Math@Mac Online Mathematics Competition
November 29, 2006

Note: All answers should be expressed as exact values where possible.

1. Two girls caught 25 frogs. Lisa caught four times as many as Jen did. How many frogs did Jen catch? Answer: 5

2. If m and n are whole numbers, 49<n<101 and 19<m<51, then what is the greatest value for (n+m)/n? Answer: 2

3. If each edge of a cube is increased by 100 percent, what is the percent increase of the surface area of the cube? Answer: 300

4. If $2^x = 15$ and $15^y = 32$, what is the value of $xy$? Answer: 5

5. What is the next number in this sequence?

   7, 33, 29, 59, 51, 86, 73, _____ Answer: 114

6. Let P be a point inside a square S so that the distances from P to the four vertices (corners), in order, are 7, 35, 49, and x. What is x? Answer: 35

7. A father in his will left all his money to his children in the following manner:

   $1000 to the first born and 1/10 of what then remains, then
   $2000 to the second born and 1/10 of what remains, then
   $3000 to the third born and 1/10 of what remains, and so on.

   When this was done, each child had the same amount. How many children were there? Answer: 9

8. Two people stand back to back next to the rails in a small railway station. As the head of the express train (which does not stop at this station) reaches them, they start to walk in opposite directions parallel to the rails. As the tail of the train reaches each of them, they stop, having walked 30 m and 40 m respectively. If they both walked with
identical, constant speeds and the train’s speed was constant, how long was the train? Answer: 240 m

9. Find the value of $s$:

$$s = 1^2 - 2^2 + 3^2 - 4^2 + \ldots - (2006)^2 + (2007)^2$$

Answer: 2015028

10. There are 120 five-digit numbers that can be formed by permuting the digits 1, 2, 3, 4, and 5 (for example, 12345, 13254, 52431). What is the sum of all of these numbers?

Answer: 3999960