This outline summarizes major points covered in lecture. It is not intended to replace your own lecture notes.

Fundamental Physical Quantities

- Length: distance or spatial separation
- Mass: quantity of physical matter (independent of weight which is a measure of gravity on mass!)
- Time: difficult to define; can use tools such as an atomic clock to define basic units of time
- Temperature: the average energy of atoms in a system
- All other physical quantities can be derived from these fundamental quantities.

Derived Physical Quantities

- Displacement (x): a change in position, specified by calculating the distance from a reference position to an ending position, and by noting the direction of movement
- Velocity (c): displacement per unit time
- Acceleration (a): change in velocity per unit time
- Force (F): the product of mass and acceleration, $F = m \cdot a$
- Pressure (P): the amount of force per unit area, P = F/A

Scalar versus Vector

- A scalar value is a measurement without direction. Example: I am 1.8m tall.
- A vector value is a measurement with direction. Example: I live 5km North West from campus.
- Simple addition and subtraction can be done with scalar but NOT vector quantities

Mass (m), Density (p) & Pressure (P)

- Mass (m) = Amount of matter present (kg, lbs)
- Density (ρ) = mass per unit volume (kg/m³, lb/in³)
- Pressure (P) = force per unit area (N/ m^2 = Pa)
- Force of gravity causes air pressure (and density) to increase toward the Earth's surface.
- Air molecules are more "compressed" at sea level than at higher levels in the atmosphere.
- The "static" atmosphere pressure is not audible; however, systematic oscillations in the surrounding air pressure may be audible.

Elasticity

- Elasticity is the tendency to resist and recover from distortion.
- Elasticity is the restoring force that allows an object to recover from a distorting force.
- Air elasticity and recovery is molecule movement that accounts for sound.
- An input force is needed for object vibration and to produce a sound.
- Newton's Laws of motion (see below)
- Hooke's Law: displacement amplitude is proportional to the applied (input) force.
- Mass and elasticity are fundamental concepts for understanding sound and vibration.
- Compression increase in density of air molecules relative to static (ambient) pressure.
- Rarefaction decrease in density of air molecules relative to static (ambient) pressure.

Newton's Laws of Motion

- Law 1: an object in a state of uniform motion tends to remain in a state of uniform motion unless an external force is applied to it.
- Law 2: relationship between mass of object (m), its acceleration (a) and the applied force (F) is: F = m⋅a
- Law 3: for every action or force (F), there is an equal and opposite reaction or force (-F).

Unit circle

- cosθ describes x-axis projected motion
- sinθ describes y-axis projected motion
- If you understand the relationship of sinθ and cosθ you can understand properties of sine waves.
- sinθ and cosθ are constant and do not vary with radius (amplitude) of unit circle (pure tone)

Sound Requires Vibration

- Any object that can vibrate can *transmit* or *receive* sound
- Any object with mass and elasticity can vibrate, and any object with mass has elasticity
- Sound can travel through anything that vibrates, not just air

Air Pressure

- Air pressure is the weight of air surrounding an object
- Air pressure provides a force on the eardrum
- Ear is very sensitive, can detect pressure changes much smaller than the static air pressure
- Static air pressure is not audible
- Due to gravity, air particles are more compressed and have a higher density closer to earth's surface

Properties of the Sound Source

- Input force is required to set an object into vibration
- Amplitude of vibratory displacement ∞ input force (Hooke's Law)

Sound and Vibrations

- Systematic vibrations of an object are transmitted to the surrounding air molecules (mass of air)
- Systematic vibrations of an object will cause local changes in air pressure and thus the density of air molecules
- Compression Increase in density of air molecules
- Rarefaction Decrease in density of air molecules

<u>Waves</u>

- For sound waves there are two types of disturbances to consider
 - 1. Rate of particle vibration (frequency of vibration)
 - 2. Rate of wave propagation (speed of sound in medium)
- Discussed two major types of wave motion
 - 1. Transverse waves: direction of particle oscillation is perpendicular to direction of wave propagation
 - 2. Longitudinal waves: direction of particle oscillation is parallel to direction of wave propagation
- Sound is a longitudinal wave

<u>Energy</u>

- Measure of the capacity to do work
- Conservation of energy: energy can be converted from one form to another but not created/destroyed
- Work a transfer of energy; a force is applied to a body causing it to move in the direction of the force
- Work = $F \cdot d$; where F = force applied to body (N) and d = distance moved by body (m)
- Units of Energy = joules (J); 1 joule = 1 N⋅m
- Potential energy (PE) versus kinetic energy (KE)
- Potential Energy (PE) = stored energy
- Kinetic Energy (KE) = energy of work
- PE + KE = 0 (Law of Conservation of Energy)
- Pendular motion: energy transfer between PE and KE; an example of simple harmonic motion
- Simple harmonic motion = uniform circular motion
- Friction provides a limit on oscillatory motion (sound energy does not travel forever)
- Equation: Angular rotation frequency is $\omega = 2\pi f$

Pendular motion

- At point of zero displacement point (bottom), KE is maximum
- At point of maximum displacement point (top; either side), PE is maximum
- Inertia causes pendulum to swing past equilibrium point
- Pendular momentum = mass x velocity (kg·m·s⁻¹)
- Total energy = PE + KE (Conservation of energy)
- Friction opposes motion and causes KE to transfer to thermal energy in medium (this is why sound does not propagate forever)

Simple Harmonic Motion

- Simple harmonic motion can be defined as projected uniform circular motion
- Uniform circular motion occurs when a body moves around the circumference of a circle at a constant rate in degrees per second (°/s)
- For any object executing uniform circular motion, we expect the angle (θ) to increase linearly with time. $\theta = \omega t$, where $\omega =$ angular rotation frequency ($\omega = 2\pi f$; f = frequency or rate of rotation) and t = time.

Time domain waveform

- Time domain: amplitude of displacement as a function of time
- Equation: $D(t) = A \cdot sin(2\pi f t + \theta)$ or $D(t) = A \cdot sin(\omega t + \theta)$, where:
- A = peak amplitude, $\omega = 2\pi f$, t = time and θ = starting phase (or position)
- Sinusoidal motion: an example of simple harmonic (uniform circular) motion

Sound Transmission

- Sound must travel through a medium.
- There is no sound in a vacuum (e.g. space).
- Amplitude of displacement is change in excursion of air molecules
- Particle displacement and particle velocity are similar to the relationship between cosθ and sinθ functions (i.e. 90° out of phase)

Sound Amplitude (A)

- Amplitude (A) of air particle displacement is proportional to applied force
- Magnitude of restoring force of elasticity is directly ∞ to magnitude of input force of displacement (Hooke's Law)
- Amplitude is a vector quantity; it has both a magnitude and a direction.
- Amplitude of a sound vibration is independent of the frequency of vibration.

Displacement (x), velocity (v), acceleration (a), pressure (Pa)

- Sound may be produced when air particles are set into vibration.
- Air particle displacement causes changes in the density of air molecules.
- Pressure is force per unit of area, hence air pressure also changes.
- Relationship between displacement (x), velocity (v), acceleration (a), pressure (Pa):
 - Air pressure and particle velocity are in phase (0° phase difference)
 - Air pressure and particle displacement are 90° out of phase (90° phase difference)
 - Air particle displacement and acceleration are 180° out of phase (180° phase difference)

Example: piston attached to a balloon

- Velocity is at a maximum when displacement is at a minimum.
- Velocity is at a minimum when displacement is at a maximum.
- Acceleration peaks when velocity undergoes the most rapid change.
- Air particle displacement changes density of surrounding air molecules.
- Air pressure and velocity are in phase.
- Air pressure and displacement are 90° out of phase.
- Air pressure and acceleration are 180° out of phase.
- Try to understand different relationships between the waveforms and how to plot them.

Describing Sound Waves:

- 1. Amplitude (A)
- Amplitude is a measure of displacement.
- Amplitude is a vector quantity.
- Unit of amplitude can vary (mm, V, etc)
- For sine waves, we measure the peak amplitude (A_{peak}) and/or the peak-to-peak amplitude (A_{peal-to-peak}).
- A_{peak} and A_{peal-to-peak} measures are good for simple sine waves, but not for complex waves.
- $A_{peak} = A$
- $A_{\text{peak-to-peak}} = 2 \cdot A$
- A more robust measure for the amplitude of <u>any wave</u> is Root Mean Square Amplitude (Arms).
- $A_{rms} = 0.707 \cdot A_{peak}$ (this relationship holdsfor sine waves only!)
- 2. Frequency (f)
- Frequency represents the number of oscillations per unit of time.
- Frequency is a scalar quantity.
- Unit of frequency is Hertz or cycles/s
- Unit originally from revolutions per minute (rpm)
- To calculate frequency, choose a time interval and count the number of events within that time interval, and then divide that count by the time interval.

Types of Damped Systems

- Damping-reduction in the amplitude (or energy) of vibrations/ oscillation.
- Damping rates contribute to duration of the signal, either high (short duration) or low (long duration).
- Undamped (lossless) system: a theoretical concept; no energy transfer (i.e. no friction present); $d_r = 0$
- Underdamped System vibrations continue for a long time; slow steady energy loss from system (e.g. tuning fork)
- Overdamped System does not vibrate or oscillate; so much friction that energy is drained too slowly from the system
- Critically Damped system does not continue to oscillate; energy decays to zero as quickly as possible without overshooting (e.g. car shock absorbers)

- 3. Period (T)
- Period is the reciprocal of frequency.
- Period is a measure of time and thus is a scalar quantity.
- Unit of period is seconds (s)
- Equation: T = 1/f
- Time to complete 1 cycle of oscillation (most useful for sine and sine-like waves)
- Frequency (f) and/or period (T) are independent of stimulus amplitude (A).
- Х
- 4. Wavelength (λ)
- Wavelength (λ) is the distance between successive condensations <u>or</u> rarefactions in a sine wave.
- Wavelength is a scalar quantity.
- Unit of wavelength is meter (m).
- Equation: $\lambda = c / f$, where c is the peed of sound in medium and f is frequency of sound.
- Constant for a given frequency, but varies for sounds traveling in different media.
- Specific to the medium because elasticity also varies with different media.
- 5. Phase (φ)
- Point in displacement (oscillatory) cycle when object begins to vibrate at specific frequency (f).
- Phase is a scalar quantity.
- Unit of phase is degrees (°) or radians (π).
- Starting phase can vary from 0° to 360° (i.e. 0 to 2π radians).
- Phase difference between two waveforms of same frequency is determined by comparing any two points along the time axis.
- Instantaneous phase difference: phase of vibration cycle at a specific point in time (or space).
- Different waveforms can be in or out of phase depending on their frequencies and starting phases.

- 6. Speed of Sound (c)
- Unit of speed of sound is m/s.
- Speed of sound is a scalar quantity.
- Sounds of different frequencies propagating through the same medium travel at the same speed.
- Frequency and speed of sound are independent variables.
- The speed at which sound propagates is characteristic of the medium.
- Speed of sound is equal to square root of elasticity (E) of medium over density (ρ) of medium.
- Equation: $c = \sqrt{(E / \rho)}$
- Speed of sound increases with increasing temperature.
- c = 331.4 + 0.607·T
- At 21°C, speed of sound in air c = 344 m/s (for comparison, the speed of light = $3x10^8$ m/s)
- Baseline formula for speed of sound was derived for 0°C in dry air (0% humidity).
- See: <u>http://en.wikipedia.org/wiki/Speed_of_sound#Practical_formula_for_dry_air</u>

Natural or Resonant Frequency (Fr) of Vibration

- Objects with mass and elasticity have potential to produce sound.
- Without friction, sinusoidal vibration is free and can be described by simple harmonic motion.
- The natural or resonant frequency of a system is the frequency at which the system most easily vibrates.
- Resonant frequency is proportional to square root of stiffness (s) of object divided by its mass (m).
- Relation: $F_r \propto \sqrt{(s/m)}$

Vibrational Damping

- Resistance and friction account for damping—the reduction in amplitude of vibration of oscillation.
- Damping rates contribute to duration of the signal, either high (short duration) or low (long duration).
- Damping factor-determined by ratio of amplitude over 2 consecutive cycles of oscillation (A_{1 &} A₂).
- Damping factor is equal to natural logarithm of ratio of A_{1 and} A₂.
- Equation: $d_f = ln(A_1/A_2)$
- Rate of damping depends on the medium.

Resonant Frequency

• The most efficient way to cause a system to vibrate maximally is to apply an input force (i.e. drive the system) at a frequency that is at or close to the natural or resonant frequency of the system.

Acoustic Impedance (Z)

- Impedance-opposition to motion
- Resistance (R) energy lost from system; motion opposition that transfers kinetic energy to heat
- Resistance of an oscillating system is frequency independent.
- Reactance (X) energy lost from system due to storage
- Units of reactance Ohms (Ω)
- Reactance of an oscillating system is frequency dependent
- KE stored as PE, and some energy lost in storage called reactance
- Two components of reactance: mass reactance and compliance reactance
- X_m = mass reactance, directly proportional to vibration frequency
- $X_m = 2\pi f \cdot m$, where m = mass
- X_c = compliance reactance, inversely proportional to vibration frequency
- $\mathbf{X}_{c} = 1/(2\pi f \cdot c)$, where c = compliance
- Resonant frequency is frequency at which mass reactance and compliance reactance cancel, and is the frequency at which the least amount of energy is required to oscillate system
- $Z = \sqrt{(R^2 + [X_m X_c]^2)}$

Below are some URLs to help you understand and further explore the relationship between the concepts of elasticity, deformation, plasticity and stiffness.

Elasticity: <u>http://en.wikipedia.org/wiki/Elasticity_%28physics%29</u> Deformation: <u>http://en.wikipedia.org/wiki/Deformation_%28engineering%29</u> Plasticity: <u>http://en.wikipedia.org/wiki/Plasticity_%28physics%29</u> Stiffness: <u>http://en.wikipedia.org/wiki/Stiffness</u> Instantaneous pressure p(t)

- Proportional to velocity of system; sound pressure is proportional to air particle velocity
- $p(t) = (m \cdot v) / (t \cdot area)$, where m=mass, v=velocity, t=time

Sound pressure and energy

- Energy-ability to do work, flows through medium
- Power-rate of energy transfer through medium, rate at which work is accomplished
- Power = Work/Time
- Sound Energy—energy unit = Joule (J) = 1 N·m
- Sound Power (W)—energy unit = Watt (W) = 1W = 1 J/s = 1 N·m/s
- Note: sound energy and sound power are proportional but are not equivalent to each other
- Sound Intensity (I)—energy unit = Watt/m² (W/m²) = 1 J/s·m²
- Sound intensity amount of energy that is transmitted per second through an area of one square meter
- Energy flowing through a volume
- Acoustical energy from point source in expanding spherical wave front

Absolute vs relative measures

- Power, intensity, pressure are absolute measures
- Relative measures used as a reference for stating relative magnitude
- Relative acoustic measures employ levels
- Levels = ratio of absolute sound energy
- Level expressed as a ratio, e.g. Power_{incident}/Power_{reference}
 - Need to know the reference or standard in order for a level measure to be useful

bel

- Number of bels calculated as: N(bels) = log₁₀(level); e.g. N(bels) = log₁₀(I_x/I_{ref})
- Bels: a relative measure without units
- Advantage: provides a smaller range of values due to compressed scale with log system
- Disadvantage: fractions commonly arise when dealing with smaller range
- Solution: use <u>decibels</u>, calculated as: $N(dB) = 10 \cdot N(bels)$; e.g. $dB = 10 \cdot \log_{10}(I_x/I_{ref})$

Intensity vs Pressure

- Sound pressure (P) is a scalar quantity; sound intensity (I) is a vector quantity
- Equation: I = [$P^2_{rms}/(\rho \cdot c)$], where ρ = density of medium and c = speed of sound in medium
- Equation: $Z_c = \rho \cdot c = characteristic impedance$ of medium

Decibels and pressure

- Standard dB equation cannot be used with sound pressure because I $\propto P^2_{rms}$
- $dB = 10 \cdot \log_{10}(I_x/I_{ref}) \rightarrow$ used for calculating decibels using absolute intensity or power
- $dB = 20 \cdot \log_{10}(P_x/P_{ref}) \rightarrow$ used for calculating decibels pressure
- ISO standard for $P_{ref} = 20 \mu Pa$
- When using the acronym dB SPL (SPL = sound pressure level) this implies $P_{ref} = 20 \ \mu Pa$

Usefulness of dB scale

- large dynamic range from threshold of hearing to threshold of pain or even hearing damage
- dynamic range can be as large 10¹² or 10¹⁴
- possible to have negative dBs; it just means that the numerator is smaller than the denominator
- Remember: once calculated, a dB is a dB is a dB

Standard dB reference

- dB SPL—sound pressure level, re 20 μ Pa
- dB IL—intensity level, re 10⁻¹² W/m²
- dB SL-sensation level, re some other measure, e.g., individual threshold
- dB HL—hearing level, re level above average threshold of hearing at that frequency
 0 dB HL at every frequency indicates subject has normal hearing (no hearing loss)
- and don't forget: a dB is a dB is a dB

Sound transmission

- Sound attenuation due to radiation of acoustic energy
- Point source of sound transmits acoustic energy in all directions as ever expanding spherical wave front
- Surface area of sphere gets larger as sound energy (power) radiates from source
- Total power (watt) of sound source equals product of sound intensity (I) and surface area at radius (r)
- Total power = $I \cdot 4\pi \cdot r^2$

Spherical spreading

- Inverse square law—energy flowing through defined surface area of sphere decreases by 1/r²
- Energy at distance 2. r from source is spread over 4 times the area and is ... 1/4 intensity
 - I = W/($4 \cdot \pi \cdot r^2$) where W = total power of sound source
- Inverse square law describes physical (geometric) spread of sound energy (i.e. sound attenuation)
- $\log(2) = 0.302$
- $\log(1/2) = -0.302$
- sound pressure changes by 6 dB for doubling/halving of distance from/to sound source

Spherical spreading: inverse square law

- <u>Relative sound intensity</u> is proportional to the reciprocal of the relative change in distance squared
- Rule of thumb: sound intensity decreases (increases) by 6 dB per doubling (halving) of distance
- Doubling absolute sound power/energy corresponds to a 3 dB change in power
- Doubling absolute sound pressure corresponds to a 6 dB change in pressure

Deviations from inverse square law

- Law is true for free, unbounded medium, but in reality there are obstacles to transfer of sound vibrational energy
- Deviations from inverse square law result from:
 - Reflection sound waves reflect off objects in path of propagation
 - o Reflection sound waves reflect off objects in path of propagation
 - Diffraction spread waves bend around objects
 - Absorption sound energy penetrates objects
 - o Refraction bending of sound waves (change of direction & speed) from one medium to another

Deviations from Inverse Square Law (Far-Field vs Near Field)

- Inverse Square Law holds true when sound pressure is measured >1-2 λ s from sound source in a free unbounded medium
- In the acoustic **far-field**, defined as >1-2 λ s from source, sound pressure > particle velocity
- Particle velocity is greatest closest to sound source (i.e. within 1-2 λ s)
- In the acoustic near-field, defined as <1-2 λs from source, particle velocity > sound pressure
- Pressure measurements very far from source may be influenced by reflected waves (i.e. reverberations or echoes)
- Boundary between near field and far field is where sound pressure and particle velocity contribute equally to measured sound intensity

Acoustic impedance (Z)

- Magnitude represents total opposition to motion
- When sound reaches boundary between different media energy may be absorbed by new medium
- Large impedance mismatch makes it difficult for the transfer of energy, so sound energy reflects
- Acoustic impedance dictates the efficiency of sound energy transfer from one medium to another

Reflection

- Law of reflection applies to both planar and spherical waves
- Reverberant waves (echoes) reflected sound waves
- Reverberation time: time for intensity of reflected sound wave to reach -60 dB re: source intensity (i.e. time for energy in reverberant waves to damp down to one millionth of source energy)

Reflections and interference

- Reflected wave can be considered as a second sound source
- Reflected waves can interact (interfere) with incident waves
 - 1. Constructive interference increase in amplitude of sound energy
 - 2. Destructive interference decrease in amplitude of sound energy
 - Phase relationship does not need to be exact for interference to occur
- Interference can result in standing wave with nodes (zero amplitude) and anti-nodes (max amplitude)

Standing wave fundamental frequency (f₀)

- Fundamental frequency (f_0) : lowest frequency that creates standing wave pattern
- Second harmonic frequency second lowest frequency of vibration (= $2 \cdot f_0$)

Reflections and standing waves

- Standing waves can occur when two waves of same frequency and amplitude travel in opposite directions (e.g. incident wave and reflected wave)
- Reflections can produce standing waves
- Resultant wave from the interaction of the incident wave and reflected wave is a standing wave that appears stationary
- Distance between two successive nodes or anti-nodes in standing wave is $\frac{1}{2}\cdot\lambda$
- Standing waves occur in fixed geometry (e.g. ear canal)
- Confined waveform propagation to a particular geometry leads to reflections and resonance
- Know which equation to use for calculating dB change (power, energy, intensity versus pressure)

Sound waves reflecting in a tube

- Fundamental frequency (f_0) is the lowest frequency (longest wavelength) that fits in tube
- Distance between successive nodes or antinodes = $\lambda/2$ = length of tube
- Wavelength (λ) of $f_0 = 2 \cdot L$ (where L = tube length)
- Higher frequency standing waves with nodes at both ends are overtones (harmonics)
 - Integer multiples of f_0 described by: $f_0 = c/2 \cdot L$; $f_2 = c/L$; $f_3 = 3 \cdot c/2 \cdot L$; $f_4 = 2 \cdot c/L$

Standing wave and resonance

- wave traveling along string is similar to geometry of standing wave in a tube
- node: point of zero displacement in standing wave (closed end of tube at f_0)
- antinode: point of maximum displacement (open end of tube at f_0)
- standing wave in tube can occur when tube closed at both ends (nodes at both ends), closed at one end and open at the other, or open at both ends (antinodes at both ends)
- tube close at one end and open at other is similar to anatomy of ear canal
- tube length (L) determines f_0 ; if you know tube geometry one can determine f_0
- $f_0 = c/2 \cdot L$ tube closed /open at both ends
- $f_0 = c/4 \cdot L$ tube closed at one end and open at other end
 - for ear canal standing wave, harmonics can occur only at odd integer multiples because those frequencies satisfy geometry (node at one end and antinode at the other end

Absorption

- Impedance change at boundary between two media (e.g. air/object interface)
- Impedance mismatch determines amount of sound energy penetration (=absorption)
- When impedance difference is infinite, all incident sound energy will reflect; when not infinite, some energy will <u>penetrate</u> and be <u>transmitted</u> to new medium
- Absorption inversely proportional to reflection

Absorption coefficient (a)

- Quantifies sound energy that penetrates or is absorbed by medium/obstacle
- Equation: $a = I_a/I_i$, where:
 - \circ I_a = intensity of sound absorbed by medium
 - \circ I_i = intensity of incident sound
- Absorption coefficient is independent of absolute sound intensity

Refraction

- Change in sound velocity (speed and direction of propagation) at boundary interface
- If sound speed in medium 1 < medium 2, then refracted wave bends toward boundary interface

Excess attenuation

- Attenuation over and above sound energy losses due to geometric radiation (spherical spreading)
- Higher frequency sounds absorbed more readily by atmosphere (by water molecules)
- Temperature and humidity affect excess attenuation measures

Complex Sounds

- Sine wave (e.g. pure tone) is fundamental component of all other waves
- Complex sounds (or waves): anything other than simple sounds (most sounds are complex signals)
- Complex waves made from the addition of simple sounds
- Type of complex sound varies with the number of simple waves added together
- Complex sound is sum of many simple sounds that differ in amplitude (A), frequency (f) & phase (ϕ).

Fourier Analysis

- Fourier analysis allows us to inspect the components that make up complex sounds
- Fourier's theorem: any complex sound can be described as the sum of sine waves of different amplitude (A), frequency (f) & phase (φ).

Graphic displays of signals

- Time domain display describes temporal (time) characteristics of signal (e.g. displacement, pressure)
- Frequency domain display describes spectral characteristics of signal (frequencies and phases)
 - 1. Magnitude Spectrum: describes amplitude of each frequency in a complex sound
 - 2. Phase Spectrum: describes starting phase angle of each frequency in a complex sound
- Both magnitude and phase spectrum necessary to <u>fully</u> characterize signal in frequency domain

Square Wave

- Series of square pulses
- Square wave magnitude spectrum: comprised of odd integer multiple harmonics of decreasing amplitude
- Phase spectrum of square wave: odd integer multiple harmonics have same starting phase

Acoustic transient

- Single square wave pulse of brief duration
- Duration (d) of transient determines where energy falls to zero on magnitude spectrum (at freq = 1/d)

Amplitude Modulation (AM)

- Two ways to create an AM stimulus with sine waves:
 - 1. Add two sine waves that are close in frequency: resultant signal has amplitude "beats"
 - Perceived frequency of the beating stimulus is $(f_1 + f_2)/2$
 - Loudness changes occur at a rate of (f_1-f_2)
 - Change in beating amplitude $(f_1-f_2)/2$
 - 2. Sinusoidal Amplitude Modulation (SAM): modulate amplitude (A) of dominant (carrier) frequency with a sinusoid of a lower frequency (modulating frequency)
 - $D(t) = A(t) \cdot sin(2\pi \cdot F_c \cdot t)$; where $A(t) = [1 + m \cdot sin(2\pi \cdot F_m \cdot t)]$
 - F_c = carrier frequency ; F_m = modulation frequency
 - Resultant signal described as: $D(t) = [1 + m \cdot sin(2\pi \cdot F_m \cdot t)] \cdot sin(2\pi \cdot F_c \cdot t)$
- Spectrum of SAM signal contains energy at sideband frequencies: $F_c F_m$ and $F_c + F_m$

Frequency Modulation (FM)

- FM is change in frequency (up or down) over time
- FM can be linear (e.g. frequency sweep) or non-linear (e.g. sinusoidal frequency modulation, SFM)
- Rate of FM can vary linearly (frequency sweep) or non-linearly (sinusoidal frequency modulation, SFM)
- Spectrogram: graphical display of change in frequency and amplitude as a function of time

<u>Noise</u>

- Created by giving a random amplitude values to signal from a predefined sampling distribution
- Amplitude Modulated (AM) noise amplitude averages to zero, but instantaneous values are random
- Noise types: narrow band, wide band
 - Narrowband Noise (NBN) relative signal bandwidth is small
 - Wideband Noise (WBN) relative signal bandwidth is large
- Bandwidth (BW) = range of frequencies signal present in signal
- Total power (TP) of noise signal: TP = $BW \cdot I$ (I = signal intensity)
- Spectrum Level (N₀) of noise signal: N₀ = TP/BW
- White noise: same average signal amplitude (spectrum level) across the entire noise bandwidth
- Pink noise: average signal power drops by -3 dB for each doubling of frequency within noise band

Resonators

- Most acoustic events generated from forced or driven vibrations
- Standing waves result as continuous forced vibrations
- Changing the number of sine wave components in a complex signal, and/or their relative amplitude(s), will cause the signal to change appearance in both the time domain and the frequency domain

Filters

- Cut-off frequency (f_c) is frequency where signal energy = $\frac{1}{2}$ input signal energy (i.e. the -3 dB frequency)
- Low pass allows frequencies below cutoff frequency to pass through unattenuated
- High pass allows frequencies above cutoff frequency to pass through unattenuated
- Band pass allows frequencies between lowpass cutoff and highpass cutoff to pass unattenuated
- Band reject attenuates frequencies between lowpass cutoff and highpass cutoffs
- A linear filter does not affect frequency component of signal, only amplitude and/or phase
- Band pass and band reject filters have both a low-pass and a high-pass cutoff frequency
- Roll-off rate: amount of attenuation (in dB) per doubling of frequency (octave)
- Roll-off rate is measured starting from the cut-off frequency (fc)
- Filter bandwidth: difference in Hz between high pass and low pas cut off frequencies (only applies for band pass and band reject filters)
- Filter bandwidth determines the pass band range of frequencies (i.e. the frequencies that pass through the filter with little or no attenuation)

Concept of Linearity

- A device is linear when it fulfills the relationships of:
 - 1. Superposition
 - 2. Homogeneity
- Superposition. A system (e.g. device) is linear when the output of a device in response to a number of independent inputs presented simultaneously is equal to the sum of the outputs that would have been obtained if each input had been presented alone
- Homogeneity. If the input to a device is changed in magnitude by a factor of **k**, then the output should also change in magnitude by a factor of **k**, but otherwise be unaltered
- The output of a linear device never contains components that were not present in the original signal