

# Bayesian Inference

**PSYCH 4KK3, PSYCH 730, Term 2, 2014**

**Mondays, 11:30 AM - 2:30 PM, MDCL 1010**

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## **Teaching Team**

Prof: Dr. Daniel Goldreich

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Office hour: Wed 5:00 - 6:00 PM (PC 413)

Teaching Assistants:

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## **Description**

This course explores the far-reaching applications in science of a simple but remarkably insightful equation. First introduced in 1764, Bayes' formula elegantly embodies the scientific method, in which predictions made from hypotheses are put to the test, and inferences regarding the relative merits of the hypotheses are drawn from the resulting data. Bayes' equation is increasingly used as a powerful research tool by scientists in all disciplines. It provides a sophisticated method for drawing inferences from data, used both for statistical analysis and as a model of human brain function. Students will apply Bayes' formula to gain insight into scientific and everyday reasoning. They will learn modern statistical analysis techniques, derived from Bayes' formula, that provide a coherent and attractive alternative to the conventional p-value methods. They will be introduced to the Bayesian foundations of signal detection theory and to Bayesian models of sensory perception. This conceptually sophisticated course is presented in an interactive and mathematically accessible fashion.

Graduate students taking Psych 730 are required to attend all lectures and complete all work along with the undergraduates in Psych 4KK3 (i.e., in-class exercises, weekly homework assignments, midterm test and final exam). The graduate students will write somewhat more advanced versions of the midterm test and final exam than the undergraduates. In addition, the graduate students will complete two additional homework assignments involving extensive reading and/or data analysis.

The course web site plays an integral role in this course. The web site contains this syllabus, links to the online course textbook, homework assignments, a password-protected system for students to check their marks, and the Bayes-Engage discussion forum. The password to the website will be provided during the first lecture.

## **Objectives**

Upon completion of this course, the student will understand:

- the concept of conditional probabilities
- how to work with the sum and product rules of probability theory
- the meaning of each element of Bayes' formula: prior, likelihood, and posterior
- key features of the scientific method that emerge from the application of Bayes' formula.
- how Bayesian inference differs from frequentist statistical methods.
- how to apply Bayesian inference to clinical diagnostic testing and genetics scenarios.
- how to use Bayesian inference to estimate the values of binomial, Poisson, and Gaussian parameters
- how to use Bayesian inference for curve-fitting and robust parameter estimation.
- how to perform Bayesian model comparison
- how to model human sensory perception as Bayesian inference

**Meetings**

Lecture: Mondays, 11:30 AM - 2:20 PM (MDCL 1010).

Tutorial (optional but strongly recommended): Wednesdays, 12:30 - 1:20 PM (MDCL 1010).

**Materials**

Required Textbook: Goldreich, D. (2013) Introduction to Bayesian Inference (online)

Students are responsible for reading each book chapter in its entirety. Some homework and exam questions may be based on material in the book that was not covered during lectures.

Note: The online book is a work in progress and may be modified slightly throughout the semester. If you are planning on printing the book, please wait until we reach the corresponding course section before printing each chapter, to ensure that you print the most recent version.

Required Calculator: The McMaster Standard Calculator (Casio FX-991) is permitted in exams and for in-class Brain Boggler exercises unless otherwise specified. This calculator is available at the McMaster Bookstore. No other calculator is permitted.

**Evaluation**

Students will be graded on their participation in the Bayes-Engage discussion forum, in-class critical thinking activities called Bayesian Brain Bogglers, weekly homework assignments, a midterm test, and a comprehensive final exam. The student's course percentage score is a weighted average of the following five items:

Item	Weight
Bayes-Engage Forum Participation	5%
Bayesian Brain Bogglers	5%
Homework Assignments	15%
Midterm Test	25%
Comprehensive Final Exam	50%

**Bayes-Engage Forum**

Students are expected to participate throughout the semester in the online Bayes-Engage discussion forum. The forum serves to facilitate student understanding of Bayesian inference by allowing students to discuss homework assignments, course material, and related Bayesian topics while not in class.

Each student is expected to contribute at least 10 substantive forum posts during the semester, and to post at least once in each of the three forum categories. A substantive post is not merely a question or a request for assistance (such posts are certainly encouraged, but do not receive credit). Rather, a substantive post is one that contributes significantly to other students' understanding of Bayesian inference. Substantive posts include: an intelligent and articulate comment about a specific concept covered in lecture or in the textbook; a correct and well-reasoned answer to another student's posted question about a challenging Bayesian topic or problem; or an astute observation regarding Bayesian inference in everyday life.

At the end of the semester, the number of substantive posts made by each student will be tallied. If a student has made at least one substantive post in each of the three forum categories, and if the student's total number of posts is greater than or equal to 10, then the student will receive a Bayes-Engage percentage score of 100%. If a student has made at least one substantive post in each of the three forum categories, but the number of substantive posts,  $n$ , is less than 10, then the student's Bayes-Engage percentage score will be  $(n / 10) * 100\%$ ; for instance, a student with 8 substantive posts will earn a score of 80%. For each category in which a student has not made a substantive post, 10% will be deducted from the student's Bayes-Engage percentage score. For instance, if a student has made 8 substantive posts, but these have occurred in two categories only, with no substantive posts in the third category, then the student's final

Bayes-Engage course percentage score will be  $80\% - 10\% = 70\%$ .

### Bayesian Brain Bogglers

The brain bogglers are in-class exercises designed to encourage critical thinking about Bayesian inference.

Some of the brain boggler exercises will require a calculator; the student should bring the McMaster Standard calculator (Casio FX-991) to class.

Typically, one brain boggler will be given per lecture; sometimes, zero or two may be given.

Unless otherwise announced, each brain boggler exercise is worth 2 points. Each answer will typically receive either zero, half, or full-credit (0, 1, or 2 points).

The student's running brain boggler total is reported online in the check-marks system.

The student's brain boggler percentage score, used in the student's course percentage score calculation, is the number of brain boggler points earned divided by the total number of points possible.

### Homework

Homework assignments are inked on the schedule table, and are due at the beginning of class each week. Late homework returns will not be accepted.

Students are encouraged to type their answers whenever possible.

Students may collaborate on homework assignments, if they wish, and collaborating students may choose to submit a single, group answer (maximum group size: 3 students). If this is done, then the name and student number of each contributing student must appear on the homework submission. Each student in the group will receive the same mark on that homework assignment. Students may change groups throughout the semester, or choose to submit their assignments individually.

The point value of each question will be indicated on the homework assignment.

The score for the entire homework assignment will always be reported on a 0-to-100% scale. For example, suppose an assignment has three questions, worth 2, 4, and 4 points each. If a student earns half credit on the first question, full credit on the second, and half credit on the third, then the student's score will be  $1 + 4 + 2 = 7$ , and the score will be reported as 70%.

The student's homework assignment scores are reported online in the check-marks system.

At the end of the term, the student's lowest homework assignment score will be dropped. The average of the student's remaining homework assignment scores will then be calculated. This is the student's course homework score.

Graduate students taking Psych 730 will complete two additional homework assignments (identified as GS HW A and GS HW B in the schedule table). These assignments, which will involve extra reading and/or data analysis, are intended to deepen the graduate students' understanding of the material covered in each of the three course sections. These assignments will be weighted equally with the other assignments in the calculation of the student's course homework score. Grad students must complete both GS HW assignments (A and B). The lowest GS HW grade will not be dropped (only the lowest of the 10 other assignments will be dropped).

### Tests and Exam

The student should bring the McMaster standard calculator (Casio FX-991) to all exams. Only the McMaster standard calculator will be allowed.

Midterm test 1 covers material from the first course segment (basic Bayesian inference and parameter estimation). Midterm test 2 covers material from the second course segment (curve fitting, robust estimation & advanced parameter estimation). The final exam is comprehensive; it covers material from all three course segments.

Graduate students taking Psych 730 will write somewhat more advanced versions of the tests and exam than the

undergraduates taking Psych 4KK3.

Each test / exam is given a mark on a scale from 0 - 100%.

The student's test marks are reported online in the check-marks system.

**Create-a-Question:** As an optional exercise, students are encouraged to try to generate an excellent exam question of their own. Please email the instructor with your proposed exam question and answer. Questions that are carefully and articulately worded, and that probe student understanding of important concepts, will be considered for inclusion. The instructor will not inform you in advance of the exam whether your question will be used, and will not provide feedback as to whether your answer is correct. If it is used on an exam, your name will not be attached to your question, but a note will be attached to indicate that the question was student-generated. Furthermore, if it is used, your question may be edited and/or otherwise modified by the instructor.

### Course Grade Calculation

The following formula is used to calculate the student's course percentage score:

Course percentage score = (course Bayes-Engage percentage score)(0.05) + (course Brain Boggler percentage score)(0.05) + (course Homework percentage score)(0.15) + (Midterm test percentage score)(0.25) + (Final Exam percentage score)(0.50)

The student's course grade will be determined from the student's course percentage score, as follows:

Letter Grade	Percentage Score
A+	90-100
A	85-89
A-	80-84
B+	77-79
B	73-76
B-	70-72
C+	67-69
C	63-66
C-	60-62
D+	57-59
D	53-56
D-	50-52
F	0-49

### Missed Work

If a student misses a lecture or a test due to illness, personal circumstances, or late registration, it is the student's responsibility to notify the instructor and to complete a McMaster student absence form, or to submit appropriate documentation (e.g. note from physician) to the appropriate Faculty/Program office. A mark of zero will be entered for all missed Brain Boggler exercises, homework assignments, and tests until the instructor receives notification from the appropriate university authority. (If a student is unable to attend a class, then the student may submit the homework assignment by email, prior to the class period, to the instructor. Late email submissions will not be accepted.) It is the student's responsibility to learn all material that the student missed for any reason; this can be done by consulting with classmates, the teaching assistants, and/or the instructor.

**Missed class periods:** If university approval is obtained for missed class periods, the total number of Brain Boggler points in the course used to calculate the student's Brain Boggler percentage score will be reduced by the number of excused Brain Bogglers that the student missed. Similarly, if university approval is obtained for missed homework, the total number of homework points in the course used to calculate the student's homework percentage score will be reduced by the number of excused homework assignments that the student missed.

**Missed Tests:** If university approval is obtained, a test missed due to illness or personal circumstances may be made up, or the course grade may be redistributed such that more weight is applied to the final exam, at the discretion of the instructor. Make-up tests, if given, may differ in format and specific content from the original test. For example, a test consisting of written questions (1-hour) and oral questions (1-hour) may be given in place of a 2-hour written original test.

### **Audio Recording**

Students may make audio recordings of the lectures, after asking permission from the instructor. Audio recordings must be used for personal study purposes only, and not posted or distributed to others in any way. Audio recordings should be destroyed when the semester ends. Photography in the classroom (taking pictures of lecture slides, or any other pictures) is strictly prohibited, as is video recording.

### **Email Policy**

Any email should state the course number and/or title in its subject heading. Please do not email the assistants or instructor with specific Bayesian inference questions; we will not answer such questions by email. Instead, specific content questions should be asked in office hours, in the tutorial sessions, or in class. In addition, students are strongly encouraged to post any such questions on the Bayes-Engage forum, where other students can help to answer them.

### **Academic Integrity**

As a student, you are expected to behave honestly and ethically at all times.

According to McMaster University's Academic Integrity Policy, you are engaging in academic dishonesty if you "knowingly act or fail to act in a way that results or could result in unearned academic credit or advantage" (Academic Integrity Policy, p. 6).

This behaviour can result in serious consequences, such as a grade of zero on an assignment, loss of credit with a notation on the transcript that reads "Grade of F assigned for academic dishonesty," and/or suspension or expulsion from the university.

It is your responsibility to understand what constitutes academic dishonesty. The following are just three forms of academic dishonesty:

1. Plagiarism.
2. Improper collaboration.
3. Copying or using unauthorized aids in tests and examinations.

For more information on academic dishonesty and academic integrity, please read the Academic Integrity Policy:  
<http://www.mcmaster.ca/academicintegrity>

### **Online Privacy**

This course makes use of an online discussion forum. You should be aware that your chosen forum username will be apparent to all other students in the course. The technology used in the course web pages is designed to keep private all additional information you provide (your name, email address, student number, and passwords). However, you should be aware that submission of such information on this or any other web site always carries some risk that the information will become public (for example, if the web site is "hacked"). Your continuation in this course will be interpreted to indicate that you consent to the disclosure of your chosen username in the online forum, and that you accept the risk of submitting the additional requested information. If you have any questions or concerns about the privacy of your information, please discuss these with the instructor.

**Weekly Schedule**

The table below shows weekly topics, assigned readings, and homework assignments (HW = homework assignment required of all students; GS HW = homework assignment required of grad students only). Students are encouraged to read the assigned material prior to each lecture. At least 12-hours prior to each lecture, the instructor will link each topic description to a PDF file containing the majority of the slides that will be shown in lecture (two PDF versions will be provided: a version with one slide per page, and a small slides version with four slides per page).

<b>Week</b>	<b>Topic</b>	<b>Homework Assignment</b>
<b>Part 1: Basic Bayes; Parameter Estimation</b>		
Jan 6	Basic Bayes: coins, apples, and other interesting items	--
Jan 13	Basic Bayes: medicine and genetics	HW 1
Jan 20	Basic Bayes: experimental design, skepticism, everyday Bayes	HW 2
Jan 27	Parameter estimation: binomial, Poisson, and Gaussian	HW 3
Feb 3	Parameter estimation: curve fitting	HW 4
Feb 10	Midterm test (2 hours)	HW 5
<b>Part 2: Adv Param Est; Model Comparison; Bayesian Brain</b>		
Feb 24	Robust estimation	GS HW A
Mar 3	Hierarchical models and advanced numerical methods	HW 6
Mar 10	Model comparison	HW 7
Mar 17	Bayesian brain: signal detection, cue combination	HW 8
Mar 24	Bayesian brain: inference from neural spike counts	HW 9
Mar 31	Bayesian brain: prediction, postdiction, and illusion	HW 10
Apr 7	Course review	GS HW B
TBA	Final exam (comprehensive, 3 hours)	

Note: The course schedule may be modified during the semester, at the discretion of the instructor. Modifications will be made directly to the table above and announced in class.