This course explores the far-reaching applications in science of a simple but remarkably insightful equation. First introduced in 1764, Bayes formula elegantly embodies the scientific method, in which predictions made from hypotheses are put to the test, and inferences regarding the relative merits of the hypotheses are drawn from the resulting data. Bayesian inference is increasingly used as a powerful research tool by scientists in all disciplines. It provides a sophisticated method for drawing inferences from data, used both for statistical analysis and as a model of human brain function. Students will apply Bayes' formula to gain insight into scientific and everyday reasoning. They will learn modern statistical analysis techniques, derived from Bayes' formula, that provide a coherent and attractive alternative to the conventional p-value methods. They will be introduced to the Bayesian foundations of signal detection theory and to Bayesian models of sensory perception. This conceptually sophisticated course is presented in an interactive and mathematically accessible fashion.

Instructor

Dr. Daniel Goldreich (goldrd@mcmaster.ca; PC 413)

Meetings

Lecture: Wednesdays, 2:30 - 5:20 PM (JHE A102).
Tutorial: Mondays, 12:30 - 1:20 PM (JHE 326H).

Graduate students taking Psych 730 are required to attend all lectures and complete all work along with the undergraduates in Psych 4KK3. The graduate students will write somewhat more advanced versions of the midterm test and final exam than the undergraduates and will complete two additional homework assignments involving extensive reading and/or data analysis.

Materials

Required Textbook: Goldreich, D. (2017) Introduction to Bayesian Inference (online)

Students are responsible for reading each assigned textbook chapter in its entirety. Some homework and exam questions may be based on material in the book that was not covered during lectures. Note: The book is a work in progress and may be modified slightly throughout the semester. If you are planning on printing the book, please wait until we reach the corresponding course section before printing each chapter, to ensure that you print the most recent version.

Required Calculator: The McMaster Standard Calculator (Casio FX-991) is permitted in exams and for in-class Brain Boggler exercises unless otherwise specified. This calculator is available at the McMaster Campus Store. No other calculator is permitted.

Objectives

Upon completion of this course, the student will understand:

- the concept of conditional probabilities
- how to work with the sum and product rules of probability theory
- the meaning of each element of Bayes' formula: prior, likelihood, and posterior
- key features of the scientific method that emerge from the application of Bayes' formula.
- how Bayesian inference differs from frequentist statistical methods.
- how to apply Bayesian inference to clinical diagnostic testing and genetics scenarios.
how to use Bayesian inference to estimate the values of binomial, Poisson, and Gaussian parameters
how to use Bayesian inference for curve-fitting and robust parameter estimation.
how to perform Bayesian model comparison
how to model human sensory perception as Bayesian inference

Weekly Schedule

The table below shows weekly lecture topics, textbook readings, and homework assignments. Students are encouraged to read the assigned material prior to the class period. Prior to each class, the instructor will post the majority of the slides to be shown in lecture. For your convenience, the slides will be posted in three PDF formats: Small (four slides per page), Lined (three slides per page, with lines on the right for taking notes), and Large (one full-size slide per page). In the table, HW = Individual homework assignment; EB = Everyday Bayes group assignment. The weekly schedule may be modified during the semester, at the discretion of the instructor. Any modifications will be announced to the class.

<table>
<thead>
<tr>
<th>Week</th>
<th>Topic</th>
<th>Assignment Due</th>
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<tbody>
<tr>
<td></td>
<td><strong>Part 1: Basic Bayes</strong></td>
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<tr>
<td>Jan 10</td>
<td>Basic Bayes: coins, apples, and other interesting items (Ch. 1)</td>
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<tr>
<td>Jan 17</td>
<td>Basic Bayes: medicine and genetics (Ch. 1)</td>
<td>HW 1</td>
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<tr>
<td>Jan 24</td>
<td>Basic Bayes: critical thinking (Ch. 1)</td>
<td>HW 2, EB 1</td>
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<tr>
<td>Jan 31</td>
<td>Parameter estimation: binomial and Poisson (Ch. 2)</td>
<td>HW 3</td>
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<tr>
<td>Feb 7</td>
<td>Parameter estimation: Gaussian (Ch. 2)</td>
<td>HW 4, EB 2</td>
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<tr>
<td>Feb 14</td>
<td>Midterm test (2 hours)</td>
<td>HW 5</td>
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<td></td>
<td><strong>Part 2: Advanced Bayes</strong></td>
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<tr>
<td>Feb 28</td>
<td>Curve fitting (Ch. 3)</td>
<td>GS HW A, EB 3</td>
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<tr>
<td>Mar 7</td>
<td>Robust estimation (Ch. 4)</td>
<td>HW 6</td>
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<tr>
<td>Mar 14</td>
<td>Hierarchical models and advanced numerical methods (Chs. 5, 6)</td>
<td>HW 7, EB 4</td>
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<tr>
<td>Mar 21</td>
<td>Model comparison (Ch. 7)</td>
<td>HW 8</td>
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<tr>
<td>Mar 28</td>
<td>Bayesian brain: perception as Bayesian inference (Chs. 9, 10)</td>
<td>HW 9, EB 5</td>
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<tr>
<td>Apr 4</td>
<td>Bayesian decision theory (Ch. 11)</td>
<td>HW 10, GS HW B</td>
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<tr>
<td>TBA</td>
<td>Final exam (comprehensive, 2.5 hours)</td>
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</tbody>
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Evaluation

Students will be graded on in-class critical thinking activities called Bayesian Brain Bogglers, weekly homework assignments, a midterm test, and a comprehensive final exam. The student's course percentage score is a weighted average of the following five items:

<table>
<thead>
<tr>
<th>Item</th>
<th>Weight</th>
</tr>
</thead>
</table>


Bayesian Brain Bogglers

The brain boggler exercises are in-class exercises designed to encourage critical thinking about Bayesian inference.

Some of the brain boggler exercises will require a calculator; the student should bring the McMaster Standard calculator (Casio FX-991) to class.

Typically, one brain boggler will be given per week, though some weeks may have more.

Unless otherwise announced, each brain boggler exercise is worth 2 points. Each answer will typically receive either zero, half, or full-credit (0, 1, or 2 points).

The student's running brain boggler total is reported online in the check-marks system.

The student's brain boggler percentage score, used in the student's course percentage score calculation, is the number of brain boggler points earned divided by the total number of points possible.

Everyday Bayes

Throughout the semester, assigned groups of students are required to submit an Everyday Bayes assignment every second week. To earn full credit, an Everyday Bayes submission must report one astute observation regarding the use of Bayesian inference in everyday life, correctly worked out mathematically with Bayes' formula.

In order to produce high-quality Everyday Bayes submissions, students are encouraged to keep a daily journal of their everyday Bayesian observations. Each student in the group must come to the group meeting with their own individual mathematically worked-out example. At the meeting, the group will then deliberate to decide which example is the best. They can also make some modifications/improvements to that example if they wish. Together, the group must then write a paragraph describing their deliberations - i.e., why did they choose the example that they did, and how (if at all) did they modify it. The group should then staple together the following pages, which must be handed in at the beginning of the class period:

1) The top page should have the example that the group decided to submit, with the name of every group member on it. This page will be graded.
2) The second page should have the one-paragraph summary description of the group's deliberations, again with the name of every group member.
3) The following pages should have each of the individual members examples that they brought to the meeting (with the name of just that individual). All group members who generated their own EveryDay Bayes example will receive the same score for the group submission. If a student did not generate their own example, they will receive a zero on the assignment.

Each EveryDay Bayes assignment will earn 5 points maximum. One point will be deducted for each error in procedure, calculation, or interpretation. Each Everyday Bayes score will be posted online in the Check-Marks system. At the end of the semester, the average of the group's Everyday Bayes scores will be taken as each
student's final Everyday Bayes score.

In order to retrieve their graded Everyday Bayes paper, the group must send at least one of its members to the prof's weekly drop-in office hour. The prof will then review the graded work with the student(s) and provide feedback on the group's work.

Homework

Homework assignments are linked on the schedule table, and are due at the beginning of class each week. Late homework returns will not be accepted.

Students are encouraged to type their answers whenever possible.

Students may collaborate on homework assignments, if they wish, and collaborating students may choose to submit a single, group answer (maximum group size: 3 students). If this is done, then the name and student number of each contributing student must appear on the homework submission. Each student in the group will receive the same mark on that homework assignment. Students may change groups throughout the semester, or choose to submit their assignments individually.

The point value of each question will be indicated on the homework assignment.

The score for the entire homework assignment will always be reported on a 0-to-100% scale. For example, suppose an assignment has three questions, worth 2, 4, and 4 points each. If a student earns half credit on the first question, full credit on the second, and half credit on the third, then the student's score will be $1 + 4 + 2 = 7$, and the score will be reported as 70%.

The student's homework assignment scores are reported online in the check-marks system. Graded homework assignments will be available for pick-up in class and in the prof's weekly drop-in office hour.

At the end of the term, the student's lowest homework assignment score will be dropped. The average of the student's remaining homework assignment scores will then be calculated. This is the student's course homework score.

Graduate students taking Psych 730 will complete two additional homework assignments (identified as GS HW A and GS HW B in the schedule table). These assignments, which will involve extra reading and/or data analysis, are intended to deepen the graduate students' understanding of the material covered in each of the two course sections. These assignments will be weighted equally with the other assignments in the calculation of the student's course homework score. Grad students must complete both GS HW assignments (A and B). The lowest GS HW grade will not be dropped (only the lowest of the 10 other assignments will be dropped).

Midterm Test and Final Exam

The student should bring the McMaster standard calculator (Casio FX-991) to all exams. Only the McMaster standard calculator will be allowed.

Graduate students taking Psych 730 will write somewhat more advanced versions of the midterm test and final exam than the undergraduates taking Psych 4KK3.

Each test / exam is given a mark on a scale from 0 - 100%. The student's test marks are reported online in the check-marks system.

Create-a-Question: As an optional exercise, students are encouraged to try to generate an excellent exam question of their own. Please email the instructor with your proposed exam question and answer. Questions that
are carefully and articulately worded, and that probe student understanding of important concepts, will be considered for inclusion. The instructor will not inform you in advance of the exam whether your question will be used, and will not provide feedback as to whether your answer is correct. If it is used on an exam, your name will not be attached to your question, but a note will be attached to indicate that the question was student-generated. Furthermore, if it is used, your question may be edited and/or otherwise modified by the instructor.

**Course Grade Calculation**

The following formula is used to calculate the student's course percentage score:

\[
\text{Course percentage score} = (\text{course Brain Boggler percentage score})(0.05) + (\text{course Everyday Bayes percentage score})(0.05) + (\text{course Homework percentage score})(0.20) + (\text{Midterm test percentage score})(0.25) + (\text{Final Exam percentage score})(0.45)
\]

The student's course grade will be determined from the student's course percentage score, as follows:

<table>
<thead>
<tr>
<th>Letter Grade</th>
<th>Percentage Score</th>
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<tbody>
<tr>
<td>A+</td>
<td>90-100</td>
</tr>
<tr>
<td>A</td>
<td>85-89</td>
</tr>
<tr>
<td>A-</td>
<td>80-84</td>
</tr>
<tr>
<td>B+</td>
<td>77-79</td>
</tr>
<tr>
<td>B</td>
<td>73-76</td>
</tr>
<tr>
<td>B-</td>
<td>70-72</td>
</tr>
<tr>
<td>C+</td>
<td>67-69</td>
</tr>
<tr>
<td>C</td>
<td>63-66</td>
</tr>
<tr>
<td>C-</td>
<td>60-62</td>
</tr>
<tr>
<td>D+</td>
<td>57-59</td>
</tr>
<tr>
<td>D</td>
<td>53-56</td>
</tr>
<tr>
<td>D-</td>
<td>50-52</td>
</tr>
<tr>
<td>F</td>
<td>0-49</td>
</tr>
</tbody>
</table>

**Policies**

**Recording**

Photographs and video recordings are strictly prohibited. Students may make audio recordings of the lectures, for personal use only and not to be posted online, emailed, distributed or otherwise shared. Students should inform the instructor in advance if they wish to make an audio recording.

**Cell Phone Use**

Cell phone use, including texting, is prohibited in the classroom. Students who need to use their cell phones should leave the classroom and return when they have finished. Cell phone use in the classroom is distracting to the professor and to nearby students, and studies have shown that students who use their phones in class learn less and obtain lower grades. For more information, see:
Laptop Computer Use

Students who use laptop computers are expected to do so for valid classroom purposes only (i.e., taking notes and displaying the posted lecture slides). Studies have shown that students who use laptops to engage in activities that are irrelevant to the course (such as social media or internet browsing) distract both themselves and their fellow students, and suffer from diminished understanding of the course material. For more information, see:

Weimer M (2012) Students think they can multitask. Here’s proof they can’t. Faculty Focus.

Email

In any email you send to the instructor or any teaching team member, please write "PNB 2XB3" in the subject line. Your email should concern logistical course issues only (e.g., to request an appointment, to ask for clarification regarding the due date of an assignment, etc.). Please do not email us with neuroscience questions; we will not answer such questions by email. Instead, specific neuroscience questions should be asked in the work/help sessions or in class. In addition, you are strongly encouraged to ask questions on the Synapse discussion forum, where other students can help to answer them.

Assignment Submission

Homework assignments are to be submitted at the front of the classroom before the start of lecture. You are encouraged to type your assignment answers, but legible handwritten answers are also acceptable. Late submissions will not be accepted. If a student is unable to attend class because of illness or transportation problems, but has completed the homework assignment, then the student should submit the homework assignment by email, prior to the start of the class period, to the instructor (goldrd@mcmaster.ca). Late email submissions will not be accepted.

Missed Work

Please see the university policy statement concerning missed work. Students who are absent for no more than three days may report their absence, once per term, without documentation, using the McMaster Student Absence Form (MSAF). Please note that the MSAF may not be used for term work worth 25% or more of the course grade, nor can it be used for the final examination. This means that the MSAF cannot be used for the midterm test or the final exam in this course. Absences of more than three days must be reported to the student's Faculty/Program office, with documentation, and relief from course work may not necessarily be granted. A mark of zero will automatically be entered for all missed work until the instructor receives notification from the MSAF system or the student's Faculty Office, and is contacted by the student to discuss how to remedy the missed work situation. It is the student's responsibility to learn all material that the student has missed for any reason. This can be done by reading the posted lecture notes and assigned textbook chapters, by consulting with classmates, and by attending office hours.

Academic Integrity
As a student, you are expected to behave honestly and ethically at all times. According to McMaster University's Academic Integrity Policy, you are engaging in academic dishonesty if you "knowingly act or fail to act in a way that results or could result in unearned academic credit or advantage" (Academic Integrity Policy, p. 6). This behaviour can result in serious consequences, such as a grade of zero on an assignment, loss of credit with a notation on the transcript that reads Grade of F assigned for academic dishonesty," and/or suspension or expulsion from the university. It is your responsibility to understand what constitutes academic dishonesty. The following are just three forms of academic dishonesty:

1. Plagiarism.
2. Improper collaboration.
3. Copying or using unauthorized aids in tests and examinations.

For more information on academic dishonesty and academic integrity, please read the Academic Integrity Policy: http://www.mcmaster.ca/academicintegrity

Note Regarding Course Dates and Deadlines
The instructor and university reserve the right to modify elements of the course during the term. The university may change the dates and deadlines for any or all courses in extreme circumstances. If such modification becomes necessary, reasonable notice will be given. It is the responsibility of the students to check their McMaster email and course websites weekly during the term and to note any changes.