Resonance flux measurement using a single-channel analyzer
and well-type NaI(Tl) detector

1. THEORY

The production of radioisotopes depends upon the characteristics of nuclear reactions. These are described by the appropriate cross-section $\sigma(E)$. At a specific energy, the reaction rate per target nucleus is the product of the cross-section and the flux density (or fluence rate) of incident particles. Reactions are never carried out at a specific energy however because the kinetic energy is distributed. This is taken into account by defining the flux density spectrum $\phi(E)$ so that the flux density for particles with energies in the interval between $E$ and $E+dE$ (or $v$ and $v+dv$) is given by

$$d\phi = n(v)vdv = \phi(v)dv = \phi(E)dE \quad (n(v): \text{number density spectrum})$$

The reaction rate is then given by

$$R = \int \phi(E)\sigma(E)dE$$

While equation (2) is generally applicable, it must be noted that in a finite sample the flux density spectrum will vary with the location of the target atom in the sample. Further discussion will be restricted to the target approximation in which this variation is considered negligible.

The most detailed consideration of equation (2) has been given to the case of radionuclide production by neutron capture reactions in which the neutrons are produced in a nuclear reactor.

(a) Neutron Capture Cross-Sections

The neutron capture cross-section exhibits strong peaks as a function of energy corresponding to resonance behaviour which results when the incident energy matches a system eigenstate. For radiative neutron capture, for which the capture of a neutron is followed by high energy photon emission, the cross-section arising from a single resonance is given by
\[ \sigma_{n\gamma} = \frac{\pi g \hbar^2}{(E - E_r)^2 + \frac{\Gamma^2}{4}} \Gamma_n \Gamma \]  

(3)

In equation (3), the quantities introduced are defined as follows:

\[ g = \frac{2 J_r + 1}{2(2 J_r + 1)} \]  

where \( J_r, J_t \) are the resonance and target spins,

\[ E_r \]  

is the resonance energy,

\[ \Gamma_n = \Gamma_n^{(o)} \sqrt{E / E_0} \]  

is the neutron width,

\[ \Gamma \]  

is the radiative width,

\[ \Gamma = \Gamma_r + \Gamma_n \]  

is the total width

\[ \h \lambda = \frac{\hbar}{2\pi} = \frac{h}{mv} \]  

is the reduced wavelength of the incident neutron

For very low energies, \( E \ll E_r \), equation (3) becomes

\[ \sigma_{n\gamma} \propto \frac{1}{\sqrt{E}} \Rightarrow \sigma_{n\gamma} = \frac{\sigma_o v_o}{v} \]  

(4)

Equation (4) illustrates the important fact that at very low energies the capture cross-section varies inversely with the speed of the neutron. By convention a standard cross-section \( \sigma_o \) is defined as the capture cross-section at the speed \( v_o = 2200 \) m/sec.

(b) The Reactor Flux Density Spectrum

The spectrum of neutrons produced by the fission reaction extends to several MeV giving rise to the fast component described by

\[ \phi_f (E) = C \sqrt{E} \ e^{-E} \sinh (\sqrt{2E}) \]  

(5)

where \( E \) is in MeV. In the reactor, collisions between the fast neutrons and light nuclei begin to moderate the neutron energy spectrum. The moderation process tends to produce a flux density spectrum

\[ \phi_r (E) = \phi_f (E) / E \]  

(6)

referred to as the resonance (or epithermal) neutron spectrum. This component is important in the energy region from ~ 0.5 eV to 10 keV.
After the moderation process ceases, neutrons are characterized by thermal equilibrium and diffuse out from the source exhibiting essentially ideal gas behaviour. The neutron density spectrum in equilibrium condition is characterized by the Maxwell’s distribution:

\[
\frac{dn}{n_t} = Bv^2 e^{-mv^2/2kT} dv \quad (n_t: \text{total density})
\] (7)

The thermal (equivalent) flux is defined as

\[
\phi_{th} = v_0 \int dn = n_v v_0
\] (8)

(c) Reaction Rates and Resonance Integrals

For a reaction induced by the reactor neutrons, the rate may be written as

\[
R = \int_0^{E_c} \phi(E) \sigma(E) dE + \int_{E_c}^{\infty} \phi(E) \sigma(E) dE
\] (9)

It is supposed that an energy \( E_c \) exists such that for \( E < E_c \) the thermal component is dominant while for \( E > E_c \) the resonance component dominates. We also consider the usual case that for \( E < E_c \) the cross-section varies inversely with speed. Then

\[
\int_0^{E_c} \phi(E) \sigma(E) dE = \int_0^{E_c} \phi(v) \sigma(v) dv
\]

\[
= \int_0^{E_c} \left( \frac{\sigma_0 v_0}{v} \right) n(v) v dv = \sigma_0 \phi_{th}
\] (10)

where \( \phi(v) = n(v) v \) is the Maxwellian flux density spectrum.

Equation (9) can now be written

\[
R = \sigma_0 \phi_{th} + I \phi_r
\] (11)

where

\[
I = \int_{E_c}^{\infty} \frac{\sigma(E)}{E} dE
\] (12)

The quantity defined in equation (12) is referred to as the resonance integral. For the case of a pure \( 1/v \) variation throughout

\[
I = \int_{E_c}^{\infty} \frac{\sigma_0 v_0}{E} dE = 2 \sigma_0 \sqrt{\frac{E_o}{E_c}}
\] (13)

The contribution from each resonance is given approximately by

\[
I_r = 2 \pi^2 g \kappa_r^2 \frac{\Gamma_\alpha \Gamma_\gamma}{\Gamma E_r}
\] (14)
which must be added to equation (13) to give the total resonance integral.

2. EXPERIMENTAL DESIGN

The measurement of the resonance flux is based on the use of a cadmium filter. The neutron absorption cross-section of Cd is extremely high for \( E < E_c \), then decreases very rapidly, so that the material can be considered to absorb all neutrons with \( E < E_c \) while being transparent to all neutrons with \( E > E_c \) (\( E_c = 0.5 \text{ eV} \)).

In this experiment identical samples of gold are irradiated under the same conditions except for the presence of a cadmium filter. The rates with and without filtering are proportional to \( I\Phi_r \) and \( R \) of eq. (11) respectively.

The relative activities produced are determined using a NaI(Tl) well detector coupled to a single-channel analyzer. This detector provides high efficiency and may be tightly shielded so that small samples may be used to meet the requirement of infinite dilution. At the same time the response is insensitive to small geometry changes, facilitating relative measurements.

3. SAMPLE PREPARATION

Four samples consisting of microgram quantities of gold are evaporated on and sealed in polyethylene sheets provided. Use a 50 \( \mu \text{l} \) aliquot of stock solution. Place two samples in each plastic vial for irradiation. Sample irradiation will then be carried out using the pneumatic delivery system in the INAA facility. Two carriers will be provided, one of which is lined with cadmium, to act as a neutron filter. Irradiations must be carried out under the direction of INAA staff.
4. EXPERIMENTAL ARRANGEMENT AND PROCEDURES

Set up the arrangement shown in the figure.

The photomultiplier tube (PMT) anode output is coupled to the preamplifier. The output of the preamp is shaped and amplified by the shaping amplifier. The single channel analyzer signal is coupled to the Y1 oscilloscope input. **The trigger source of the oscilloscope is the Y1 signal during the experiment.**

Another branch of the amplifier output is delayed by 2 μsec with the external delay box before being presented to the Y2 input.

To set up the system, insert the source in the well, and apply the high voltage (100 V step, Do NOT exceed 1,000 V !!) to the PMT until you can get a good signal amplitude. Set the oscilloscope to trigger positively on the Y2 input, at 1 or 2 μsec/cm sweep rate. You should now obtain triggered operation, displaying the entire pulse spectrum. Vary the high voltage until an intense pulse appears in the spectrum. This corresponds to the full-energy deposition.

Set the oscilloscope to trigger as in the figure and observe the effect of varying the single-channel analyzer threshold and width. Also vary the amplifier gain controls, and observe the effect.

Perform the following measurements:

1) Calibrate the single channel analyzer by determining the threshold setting which produces maximum intensity for the set of calibration sources provided. $[^{114m}\text{In} \ (190 \text{ keV}), \ ^{198}\text{Au} \ (411 \text{ keV}), \ ^{137}\text{Cs} \ (662 \text{ keV}), \ ^{60}\text{Co} \ (1173, 1332 \text{ keV})]$
2) Place one of the gold samples in the test tube provided, and set the single channel analyzer to accept the pulse-height region encompassing the full energy peak at 0.411 MeV. Connect the analyzer output to the signal input of the counter unit. Determine the relative activities of all gold samples. The time of measurement should be recorded as well.

5. QUESTIONS

(1) Explain briefly how the oscilloscope arrangement functions. In particular, why is the 2 µsec delay introduced between the amplifier output and the oscilloscope input?

(2) Make a graph of the threshold setting against gamma-ray energy using the calibration data.

(3) Calculate $^{197}$Au resonance integral from equations (13) and (14) arising from the lowest energy resonance with parameters:

$$E_r = 4.906 \pm 0.01 \text{ eV}, \quad \Gamma = 139 \pm 3 \text{ meV},$$

$$\Gamma_n = 15.6 \pm 0.5 \text{ meV}, \quad \Gamma_\gamma = 124 \pm 3 \text{ meV},$$

$$J_r = 2^+, \quad J_t = 3/2^+$$

(4) Calculate the ratio of resonance flux density constant $\phi_r$ to thermal flux density $\phi_{th}$ from your measured relative activities using $\sigma_0 = 98.8 \pm .3b$ and I from question (3).