Chapter 2  General Properties of Radiation Detectors

Ionizing radiation is most commonly detected by the charge created when radiation interacts with the detector. The definition is, after all, *ionizing radiation*. Crudely, the efficiency with which a detector measures a particular type of radiation depends on the efficiency with which the radiation type creates charge within the detector.

When charged particles like protons or electrons are incident on the detector, they continuously interact with atomic electrons of the detector material and produce electron-ion (or electron-hole) pairs along their tracks. This initial charge produced by radiation is most important information and further processed by a proper signal processing electronics after collection.

When neutral radiation fields like those consist of gamma-rays or neutrons are measured, a neutral particle interacts with the detector material and then the product secondary charged particle (electron, proton, alpha etc.) deposits its energy in the detector.

2.1. Operation modes

Assume that a radiation interaction with a detector created charge $Q$ within the detector volume. As the collection process of charge $Q$ progresses, a current signal $i(t)$ is induced at the collection electrode.

\[
\int_{0}^{t_c} i(t) dt = Q
\]

$t_c$: charge collection time

The detection event rate (or counting rate) is dependent on both radiation field and detector efficiency. If the event rate is not high, each detection event can be well separated and analyzed.
A. Current mode
A simple way of measuring a detector signal is current measurement. In this mode, a current meter is connected to the detector output. Since the current level is in pA or nA, a precise meter is required. Given that the current meter has a response time $T$, the observed current from a sequence of events at time $t$ will be

$$
\bar{i}(t) = \frac{1}{T} \int_{t-T}^{t} i(t') dt'
$$

The response time is usually longer than the time between individual detection events, so that an average current is recorded at a time $t$. The current mode is used when event rates are very high, which makes a stable current.

B. Pulse mode
In the pulse mode, we preserve the information on energy and timing of individual events, i.e. the information on the signal amplitude and time of occurrence is preserved. The signal shape from a radiation detector depends on the electronics to which the detector is connected as well as the detector response. Often, the input stage of the electronics is an RC circuit.

In the circuit, $R$ represents the measuring circuit input resistance, $C$ is the summed capacitance of the detector, the cable and the input capacitance of the preamp. $V(t)$ is the time dependent voltage across the load resistor; $V(t)$ is the signal that is produced. $\tau = RC$ is the time constant of the measuring circuit.

![Fig. 2.1. Assumed output signal and signal voltage V(t).](image)
There are two extremes of operation: small RC \((\tau << t_c)\) and large RC \((\tau >> t_c)\). Circuits with large RC are common in **radiation spectroscopy** systems, in which the energy distribution of the incident radiation is measured. Many of the detector types used in the 4R6 laboratory will have output pulses of roughly the shape described in Fig. 2.1 when viewed on an oscilloscope.

With the large RC circuits, the maximum voltage becomes

\[ V_{\text{max}} = \frac{Q}{C}, \]

where, \(Q\) is the charge produced in the detector. If \(C\) is fixed and stable, \(V_{\text{max}}\) is directly proportional to \(Q\). Therefore, **measuring the pulse height** \(V_{\text{max}}\) **is equivalent to measurement of the charge** (or deposited energy) produced by a radiation interaction.

### 2.2. Energy resolution

![Energy resolution example](image)

**Fig. 2.2. Examples of good and poor energy resolutions.**

The energy resolution of a radiation detection system is a most important property when radiation spectroscopy is intended. Fig. 2.2 shows pulse height spectra from two detection systems for the same radiation source. When a monoenergetic source is measured and each system produces the corresponding response of a simple peak, the system with a good resolution gives a narrower peak width, which is beneficial for separating closely located peaks. In an ideal case, a delta function type resolution would be the best case, however, a real radiation spectroscopy system always produces a finite energy width in the peak shape. The energy resolution depends on the type of the detector and the configuration of the noise filtering in pulse processing.
When mono-energetic radiation is incident on a detector and the energy (or pulse height) spectrum is measured, the peak produced by full-energy deposition is Gaussian as shown in Fig. 2.3 if the number of counts is sufficient enough. The energy resolution of a peak can be expressed as the width of the peak. Thus, a good resolution means narrower peak width. A conventional way of defining the width is **Full Width at Half Maximum** (FWHM). From the definition of Gaussian function

\[ G(E) = \frac{A_p}{\sigma \sqrt{2\pi}} \exp\left(-\frac{(E - E_0)^2}{2\sigma^2}\right) \]

where, \( A_p \): peak area or number of counts, \( \sigma \): standard deviation, \( E_0 \): peak center (or the energy of the incident radiation), FWHM has a relation of

\[ \text{FWHM} = 2.355\sigma \]

with the standard deviation. One of the origins of the peak width is the statistical fluctuation in the number of charge carriers produced by radiation interaction. If this process follows a Poisson distribution, the standard deviation is the square root of the number of charge carriers. Since the number is proportional to the \( E_0 \), the peak width has a dependence of \( \sqrt{E_0} \) and becomes broader as the incident radiation energy \( E_0 \) increases.

When a fractional instead of absolute value is required, the percent resolution can be defined as

\[ R = 100 \times \frac{\text{FWHM}}{E_0} \]

For example, if a gamma-ray spectrum shows a peak with 2 keV FWHM at 1332 keV, the percent resolution at this energy becomes 0.15 %.

### 2.3. Detection efficiency

Detection efficiency is another important property of a radiation detector. As the general meaning implies, detection efficiency represents the probability of detection for a single radiation quantum. An accurate and precise calibration of detection efficiency is very important for quantitative measurement of an unknown radiation source and will be discussed in detail in the photon spectrometry chapter.
An ideal radiation detection system should have a high efficiency and a good energy resolution, which is hardly met in practical applications. Therefore, when you plan to set up a radiation detection system, the detector type and material should be carefully chosen according to the priority of the measurement. A compromise is usually unavoidable.

In general, detection efficiency is dependent on both radiation interaction and size of a detector. Charged particles (electron, proton, alpha) interact more easily than neutral ones (X-ray, gamma-ray, neutron) and give high efficiencies.

Theoretically, if we could create detectors with large enough volumes, we could always detect 100% of the particles incident on the detector. However, this is either impractical or even impossible, e.g. semiconductor crystals just cannot be grown large enough to be 100% efficient for high energy photons. Neutrino detectors are already built in mineshafts. This is why the concept of detection efficiency was created. Not all particles can be detected, but if the proportion of detected particles is known, the number of particles can be calculated from the number detected.

Depending on the way of defining the number of radiation quanta, either absolute or intrinsic efficiency can be used. The definition of the absolute efficiency is

$$\varepsilon_{\text{abs}} = \frac{\text{Number of pulses detected}}{\text{Number of radiation quanta emitted from source}}.$$  

For example, if we have a radiation source emitting \( Y \) particles per second, \( \dot{C} \) is the measured counting rate, and we know \( \varepsilon_{\text{abs}} \), we can calculate \( Y \) from

$$Y = \frac{\dot{C}}{\varepsilon_{\text{abs}}} \quad [\text{particles/s}]$$

A shortcoming of the absolute efficiency is that it changes every time when the detection geometry is changed, for example, the source or the detector is relocated to a different position. To make the efficiency almost independent of the detection geometry, the intrinsic efficiency was introduced:

$$\varepsilon_{\text{int}} = \frac{\text{Number of pulses detected}}{\text{Number of radiation quanta incident on detector}}$$

Now, the efficiency is defined per number of radiation quanta incident on the detector, so that the influence of the detection geometry is much relieved and the efficiency is nearly independent of the geometry. However, at very short distances between detector and source, even the intrinsic efficiency may vary significantly due to variation in the path length distribution and therefore, care must be taken in this case.

The intrinsic and absolute efficiencies can be related to each other by the probability of incidence on the detector. The solid angle is used to represent this probability. The definition of the solid angle is the fraction of a specific angular range (both polar and azimuthal angles) in 3-dimensional space. There are two units used for the solid angle. The fractional solid angle makes a sphere (i.e. all direction) 1 while the steradian [sr] makes \( 4\pi \). The fractional solid angle has a
meaning of probability. Both solid angle units can be converted by multiplying or dividing by \( 4\pi \). Practically, the steradian unit is mostly employed. Thus, \( \varepsilon_{\text{int}} \) can be converted into \( \varepsilon_{\text{abs}} \) by

\[
\varepsilon_{\text{abs}} = \varepsilon_{\text{int}} \frac{\Omega}{4\pi}
\]

when the solid angle is given in [sr].

Suppose an angular range is defined in \((\theta_1, \theta_2), (\phi_1, \phi_2)\) intervals (\(\theta\): polar, \(\phi\): azimuthal angles). This angular range will make a surface area of \(S\) on the surface of a sphere with radius \(r\) as

\[
S = \int_{\phi_1}^{\phi_2} \int_{\theta_1}^{\theta_2} r^2 \sin \theta \ d\theta d\phi
\]

Since the solid angle is proportional to the surface area but has to be independent of the radius, it can be defined as

\[
\Omega = 4\pi \frac{S}{4\pi r^2} = \int_{\phi_1}^{\phi_2} \int_{\theta_1}^{\theta_2} \sin \theta \ d\theta d\phi = \int_{\phi_1}^{\phi_2} \int_{\theta_1}^{\theta_2} d\Omega \ [\text{sr}]
\]

in steradian. In general situation, the solid angle can be calculated by integrating the differential solid angle \(d\Omega\) over given angular intervals.

As a simple example, suppose a point source and a cylindrical detector with radius \(a\). The solid angle in this case becomes

\[
\Omega = \int \sin \theta \ d\theta d\phi = 2\pi (1 - \frac{d}{\sqrt{d^2 + a^2}}) \ [\text{sr}]
\]

When the distance \(d\) is close to 0, the solid angle becomes \(2\pi\), which is equivalent to that of a hemisphere. If the distance is much longer than the radius \(a\), the spherical surface area can be approximated to the cross-sectional area \(\pi a^2\) and the solid angle becomes

\[
\Omega = \frac{S}{r^2} \approx \frac{\pi a^2}{r^2} \ [\text{sr}]
\]

Efficiencies are also classified by the fraction of the energy deposited. Depending on interactions involved with each radiation particle, there are two possibilities of energy deposition: full or partial energies of the incident radiation. If we don’t care how much fraction of energy is deposited, i.e. accept all pulses from the detector, the total efficiency is used. The full energy deposition events form a peak in the energy spectrum as shown in Fig.2.3 and the peak efficiency is defined with full energy events.

The detector efficiency should be specified according to both criteria. For example, the conventional format used for gamma-ray detectors is the intrinsic peak efficiency.
2.4. Dead time

Typical pulse processing systems for radiation detectors are shown in Fig. 2.4. The first one is for radiation spectroscopy while the second is a simple counting system with a Geiger-Müller (GM) gas detector. In the first case, radiation is incident on the detector, where charge is created. A voltage pulse is passed to the amplifier where it is shaped and amplified. The pulse is then passed to the MultiChannel Analyzer (MCA) where its height is digitized. All of these processes take time. While one pulse is being processed, another event cannot be. The time this takes is called the dead time.

The dead time of a system is the summation of all the processing times of the different components - detector, amplifier, MCA. In the laboratory, you will measure the dead time in a GM tube. This is a very simple system, as shown above, and essentially you will be looking at the dead time of the GM detector itself; the time that the GM tube needs to process a pulse.

If dead time losses are not accounted for, this can lead to misleading results e.g. source activities will be underestimated. There are two models for dead time behavior: paralyzable (or extending) and non-paralyzable (or non-extending). These are idealized responses that predict extreme behaviour. True systems, being a combination of components, will often be somewhere between the two models.

Fig. 2.5. Models of dead time behavior.
A. Non-paralyzable (non-extending) model

A fixed dead time $\tau$ follows each event that occurs during the live period of the detector. Events that occur during the dead period are not recorded and have no effect on the system. In Fig. 2.5, we end up recording only 4 counts instead of 6 real events.

Assume we have a detector system with a steady state source e.g. an extremely long-lived radioisotope and let’s define $\dot{C}_n$ = true event rate, $\dot{C}_m$ = recorded count rate, $\tau$ = system dead time. In 1 s interval, we have $\dot{C}_m$ counts (each count with a time width of $\tau$) and the event loss probability becomes $\tau \dot{C}_m$. Accordingly, the rate of the loss events is expressed as $\tau \dot{C}_n \dot{C}_m$. The rate of event loss is also $\dot{C}_n - \dot{C}_m$. Therefore,

$$\dot{C}_n - \dot{C}_m = \tau \dot{C}_n \dot{C}_m \Rightarrow \dot{C}_n = \frac{\dot{C}_m}{1 - \tau \dot{C}_m} \quad (or \quad \frac{1}{\dot{C}_m} = \tau + \frac{1}{\dot{C}_n})$$

B. Paralyzable (extending) model

In this model, a fixed dead time $\tau$ also follows each event during the live period of the detector. However, events that occur during the dead period, although not recorded, still create another fixed dead time $\tau$ on the system following the lost event. In Fig. 2.5, we end up recording only 3 events rather than 6 true events. In extreme cases, where the count rate is high, we can end up switching off the system, as pulses and dead times overlap and we record no events, hence the term paralyzed. The dead periods are now not always of a fixed length, so the true event rate obtained in the non-paralizable model is not effective here.

Before deriving the dead time, we need to investigate the distribution function for time intervals between successive events. From statistics, the probability of the binomial distribution is given as

$$P(x) = \frac{n!}{(n-x)!x!} p^x (1-p)^{n-x},$$

where, $n$ is the number of trials, $p$ is the success probability per trial, $x$ is the number of events occurred. The mean value and the variance of the distribution are $\mu = np$ and $\sigma^2 = np(1-p)$, respectively. As an extreme case, if the probability $p$ is very small, the binomial distribution reduces to

$$P(x) = \frac{\mu^x e^{-\mu}}{x!},$$

which is the Poisson distribution. The Poisson distribution can be conveniently adopted to describe the time interval distribution between successive events of a radiation detector if general requirements are satisfied.

Assuming an event has occurred at time $t = 0$, the differential probability that the next event will occur at $t = t'$ within a differential time $dt'$ is

$$P_1(t')dt' = \frac{\text{Probability of next event taking place at } t=t' \text{ within } dt'}{\text{Probability of no event in } (0, t') \text{ interval} \times \text{Probability of an event during } dt'}$$
From the Poisson distribution, \( P(0) \) becomes
\[
P(0) = \frac{(\hat{C}_n t')^0 e^{-\hat{C}_n t'}}{0!} = e^{-\hat{C}_n t'},
\]
and accordingly,
\[
P(t')dt' = \hat{C}_n e^{-\hat{C}_n t'} dt'.
\]
The probability of observing an interval larger than \( \tau \) can be obtained by integrating
\[
\int_\tau^{\infty} P(t')dt' = e^{-\hat{C}_n \tau}
\]
The rate of such intervals is then obtained by multiplying with the true rate
\[
\hat{C}_m = \hat{C}_n e^{-\hat{C}_n \tau}
\]
There is no explicit solution for \( \hat{C}_n \), it must be solved iteratively to calculate \( \hat{C}_n \) from measurements of \( \hat{C}_m \) and \( \tau \).

A plot of the observed rate \( \hat{C}_m \) as a function of the true rate \( \hat{C}_n \) is given in Fig. 2.6 for a dead time \( \tau \) of 200 \( \mu \)s. When the event rates are low, the two models give the same results, however, the trends are totally different in high rates. The nonparalyzable model approach an asymptotic value of \( 1/\tau \). For paralyzable model, a maximum value is formed at \( \hat{C}_n = 1/\tau \) and then the observed rate decreases as the true rate increases. Thus, the observed rate can correspond to either a low true rate or a high rate as shown in the figure. In practice, any ambiguity is solved by varying the count rate up or down and observing whether \( \hat{C}_m \) increases or decreases. Since the dead time models are imperfect, the dead time of a radiation detection system is generally set at < 20 \%.

![Fig. 2.6. Observed rates as a function of the true rate for two dead time models.](image-url)
2.5. Limits of detection [4,5]

A. Critical level, $L_C$

Suppose a radiation detector is given and we are measuring the number of detection events. If a radioactive source is positioned and we take a measurement for a certain counting time, the detection system will produce a number of counts (or detected events) $C_G$. Since both radiation from the source and background radiation contribute to the gross counts $C_G$, the net count $C_N$ can be expressed as

$$C_N = C_G - C_B$$

where $C_B$ is the average number of counts from the background radiation.

If there is no radioactive source or sample, the observed counts are from the background radiation only. Suppose we take measurements a large number of times with no source at a fixed counting time. A series of background counts would be obtained and the net count $C_N$ will be distributed as shown in Fig. 2.7. Since there is no source, the mean value of the net count is zero. The standard deviation of the distribution is $\sigma_0$ as shown in the figure.

Then we have the following question: how can we decide whether a measured net count from an unknown sample close to zero is truly zero or is a true existing count from a source? The critical level $L_C$ is the answer to this question and is defined as the count above which we can assume that a measured net count is meaningful. The critical level can be set at

$$L_C = k_\alpha \sigma_0$$

as shown in Fig. 2.7. In other words, if a measured $C_N$ is bigger than $L_C$, we will conclude that the true detection events “may exist”. Here, the constant $k_\alpha$ corresponds to the degree of confidence (or risk of mistake, $\alpha$) as in hypothesis testing. In our case, we don’t care about the region of $C_N < 0$, so that one-sided testing is applied. For example, if the risk is set at 0.05 (or a confidence level of 95%), the corresponding $k_\alpha$ value is 1.645 for a Gaussian distribution. Depending on how much confidence level you want, $k_\alpha$ can be set differently.

<table>
<thead>
<tr>
<th>Risk of mistake, $\alpha$</th>
<th>One-sided degree of confidence</th>
<th>$k_\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.01</td>
<td>99.0%</td>
<td>2.326</td>
</tr>
<tr>
<td>0.05</td>
<td>95.0%</td>
<td>1.645</td>
</tr>
<tr>
<td>0.10</td>
<td>90.0%</td>
<td>1.282</td>
</tr>
</tbody>
</table>
From the definition of the net count, the variance of the net count is represented as
\[ \text{var}(C_N) = \text{var}(C_G) + \text{var}(C_B) = C_G + C_B = C_N + 2C_B \]

Then the standard deviation \( \sigma_0 \) can be obtained from
\[ \sigma_0^2 = \text{var}(N_C = 0) = 2C_B \]
and the critical level becomes
\[ L_C = k_\alpha \sigma_0 = k_\alpha \sqrt{2C_B} \]

**B. Detection limit, \( L_D \)**

The detection limit is the answer to the question of “**What is the minimum number of counts we can be sure of detection?**”. The critical level is not a strong indication of true detection as already discussed. If a measured count is exactly \( L_C \), we can only be sure of true detection in 50\% of cases since the counts will be distributed symmetrically relative to \( L_C \) when many measurements are taken. Therefore, it is straightforward that the detection limit should be above \( L_C \).

Suppose a radioactive sample produced a count exactly at the detection limit of the measurement system with a standard deviation of \( \sigma_D \). If we would like to control the risk of not detecting this sample at \( \beta \), the detection limit \( L_D \) is defined by
\[ L_D = L_C + k_\beta \sigma_D = k_\alpha \sigma_0 + k_\beta \sigma_D \]
as shown in Fig. 2.8. For convenience, if \( \alpha \) and \( \beta \) are set at the same risk level and accordingly, the constants \( k_\alpha, k_\beta \) are same, the detection limit is simplified to
\[ L_D = k \sigma_0 + k \sigma_D \]

The variance of the distribution \( \sigma_D^2 \) is
\[ \sigma_D^2 = C_G + C_B \]

At the detection limit, the gross count \( C_G \) is \( C_G = L_D + C_B \) and the background count \( C_B \) has a relation \( 2C_B = \sigma_0^2 \) as already shown. With these relations, the standard deviation \( \sigma_D \) is represented by
\[ \sigma_D^2 = L_D + \sigma_0^2 \]

Then, the detection limit equation becomes
\[ L_D = k \sigma_0 + k \sigma_D = k \sigma_0 + k \sqrt{L_D + \sigma_0^2} \]
the solution of which gives
\[ L_D = k^2 + 2k\sigma_0. \]

In most cases, the \( k^2 \) term is relatively much smaller and therefore, \( L_D \) is finally simplified to

\[ L_D \approx 2k\sigma_0 = 2k\sqrt{2C_B} \ [\text{counts}] \]

For example, if a risk of 5% (or 95% confidence level) is adopted, the detection limit of the system becomes \( 4.65\sqrt{C_B} \). The Occupational Nuclear Medicine Group at McMaster has been developing in vivo analysis methods for trace elements in body and has employed \( 2\sigma_0(=2.83\sqrt{C_B}) \). A conventional definition of \( 3\sqrt{C_B} \) has been used in the radioanalytical nuclear chemistry community.

Since the detection limit is proportional to the square root of the background count, we should maintain the background counting rate as low as possible by adding appropriate shielding for the background radiation.

The dimension of the detection limit can be converted to radioactivity of the sample

\[ L_D \ [\text{Bq}] = \frac{L_D \ [\text{counts}]}{t_c [s]} \frac{1}{\epsilon_{abs} p_e} \]

where, \( p_e \) is the emission probability of the radiation per decay (note: for gamma-rays \( p_e \) is generally lower than 1). If decay of radioactivity is negligible during the counting period and the background counting rate is constant, the detection limit in [Bq] is inversely proportional to \( \sqrt{t_c} \). Therefore, we can improve the detection limit by counting longer.

References

**Problems**

1. Derive the Poisson distribution from the binomial distribution.

2. For a Gaussian function, derive the full width at the **fifth maximum** in terms of the standard deviation.

3. Derive the solid angle formula for a point source and a cylindrical detector.

4. Suppose you counted the background radiation many times without any radioactive samples, which gave a standard deviation of $\sigma_0$ for the “net counts”.
   (a) Using the Gaussian probability density function $G(x) = \frac{1}{\sigma \sqrt{2\pi}} \exp\left(-\frac{(x-x_0)^2}{2\sigma^2}\right)$, write down a mathematical expression for the degree of confidence in the case the critical level is set at $L_c = k\sigma_0$.
   (b) Express the degree of confidence for $k = 1$ using the complementary error function $\text{erfc}(x)$.

5. A radiation detection system showed the detection limit of 20 counts for 1 hour counting at 95\% degree of confidence.
   (a) Find the counting rate of the background radiation.
   (b) Find the detection limit for 3 hour counting at 90\% degree of confidence.

6. A radiation detection system has a mean background counting rate of 2 counts per second. A 30 min counting for an unknown radioactive sample led to the gross counts of 4,000 counts.
   (a) Compute the net count and its uncertainty.
   (b) Find the minimum counting time required to reach 5\% uncertainty for the net counts.

7. A 30 min counting for an unknown radioactive sample led to the gross counts of 4,000 counts. The background counts for the same counting time were 3,600 counts.
   (a) Compute the net count and its uncertainty.
   (b) Find the minimum counting time (hours) required to reach 5\% uncertainty for the net counts.

\begin{table}[ht]
\centering
\begin{tabular}{|c|c|c|}
\hline
Risk of mistake, $\alpha$ & One-sided degree of confidence & $k_{\alpha}$ \\
\hline
0.01 & 99.0 \% & 2.326 \\
0.05 & 95.0 \% & 1.645 \\
0.10 & 90.0 \% & 1.282 \\
\hline
\end{tabular}
\end{table}
8. A radiation detection system has an average counting rate of 5 counts per second. Supposing a detection event happened at time $t=0$.
(a) Find the mean number of counts for an interval $(0, t_1)$. Find the probability of zero detection in this interval.

(b) Find the probability that the next detection event will happen in the time region $(t_1, t_2)$. Compute the probability for the case $t_1 = 0.1$ s and $t_2 = 0.5$ s.

9. A point radioactive source was placed at 10 cm from a cylindrical detector with 100 $\mu$s dead time and the observed counting rate was $2.4 \times 10^5$ counts/min.
(a) Using the non-extending (non-paralyzable) model, find the fraction of the counting loss, i.e. lost events to true events ratio.

(b) Estimate the “observed counting rate” when the source-to-detector distance is doubled.

10. A radiation detector undergoes a fixed dead time of 100 $\mu$s per each detection event. Using the extending (paralyzable) dead time model $\dot{C}_m = \dot{C}_n e^{-\tau e}$,
(a) Find the percentage dead time, i.e. dead rate to true rate ratio, for a true event rate of 30,000 counts/min.

(b) Sketch $\dot{C}_m$ as a function of $\dot{C}_n$ in the $\dot{C}_n$ region of 0 to 500 counts/s. Briefly explain the reason why $\dot{C}_m$ has the sketched trend in this region.

11. To determine the dead time of a radiation counting system, a set of radioactive sources with various activities were produced and their counting rates were measured. Using the extending (i.e. paralyzable) dead time model, the following relation was built by fitting the experimental data:
\[
\ln(\frac{\dot{C}_m}{x}) = 2.303 - 0.002x \quad (\dot{C}_m: \text{measured counting rate [counts/s]}, \quad x = \dot{C}_n / \dot{C}_{n,1}, \quad \dot{C}_n: \text{true interaction rate}, \quad \dot{C}_{n,1}: \text{true interaction rate for the weakest source used})
\]
Find the dead time.

12. A radiation detection system showed the detection limit of 100 counts for an hour counting time. The corresponding detection limit in Bq was 10 Bq. Write down the expected detection limits in [counts] and [Bq] for a four hour counting. Briefly explain the reason.

13. A point radioactive source was placed at 10 cm from a cylindrical detector with 100 $\mu$s dead time and the observed counting rate was $2.4 \times 10^5$ counts/min. Using the extending (paralyzable) model, find the true interaction rate in $[s^{-1}]$ at which the observed counting rate is maximum. (10)