Chapter 6 Pulse Processing

6.1. Introduction

Most radiation detectors require pulse (or signal) processing electronics so that energy or time information involved with radiation interactions can be properly extracted. The objective of this chapter is to study and understand the general principles of pulse processing in radiation detection. There are two types of signal pulses in radiation measurements: linear and logic pulses. A linear pulse is a signal pulse carrying information through its amplitude and shape. A logic pulse is a signal pulse of standard size and shape that carries information only by its presence or absence. Generally, linear pulses are produced by radiation interactions and then converted to logic pulses.

It has become common practice to manufacture most pulse processing electronics in standard modules so that they can fit into a housing called a bin or crate, which occupies 19 inch width. Most popular standards are the Nuclear Instrument Module (NIM) and Computer Automated Measurement and Control (CAMAC). Commercial modular electronics are usually manufactured according to one of these standards. The convenient features of these standards are:

- The basic dimensions for the bin and modules are specified.
- Only the bin is connected to the ac power and generates all the dc supply voltages required by modules contained within that bin.
- The connector interface between the module and bin is standardized both electrically and mechanically.
- Specifications are included for the polarity and span of both linear and logic pulses.

6.2. Common components

A. Preamplifier

A preamplifier is the first component in a signal processing chain of a radiation detector. The charge created within a detector is collected by the preamp. In spite of its name, the preamp does not act as an amplifier (just means “before”, i.e. “pre” the amplifier), but acts as an interface between the detector and the pulse processing electronics that follow. The main function of a preamplifier is to extract the signal from the detector without significantly degrading the intrinsic signal-to-noise ratio. Therefore, the preamplifier is located as close as possible to the detector, and the input circuits are designed to match the characteristics of the detector. Two important requirements of the preamp are:

- To terminate the capacitance quickly to maximize the signal-to-noise ratio.
- To have a low output impedance; i.e. to provide a low impedance source for the amplifier. Of course, it should provide a high impedance load for the detector.

The schematic diagram of a RC feedback charge sensitive preamp is shown in Fig. 6.1. The detector high voltage bias is fed through the preamp in general except for scintillation detectors. When a preamp is ac-coupled as shown in Fig. 6.1, a single cable is connected between the preamp and the detector, and is used for both high voltage bias to the detector and signal...
extraction from the detector. A coupling capacitor should be provided between the detector and preamp circuits in this configuration while it is eliminated if the dc-coupled configuration is adopted instead.

In the **charge sensitive preamp**, charge from the detector is collected on the feedback capacitor $C_f$ over a period of time, effectively integrating the detector current pulse. As the charge is collected, the voltage on the feedback capacitor rises, producing a step change in voltage. The output voltage is then proportional to the total integrated charge as long as the time constant $R_f C_f$ is sufficiently longer than the duration of the input pulse. In other words, the output pulse height is in proportion to the energy deposited by a radiation interaction. As shown in Fig. 6.2, the output pulse shape is characterized by a short rise time determined by the charge collection characteristics of the detector, and long decay time (~ 100 $\mu$s).

In normal operation at ordinary counting rates, the rising step caused by each detector event rides on the exponential decay of a previous event due to the long decay time, and the preamp output does not get a chance to return to the baseline. This does not create a serious problem since the significant information of the output pulse is in its rising edge and the shaping amplifier is capable of extracting the pulse height from the rising edge of each pulse. As the counting rate increases, the pile up of pulses on the tails of previous pulses increases, and the excursions of the preamp output move farther away from the baseline. The dc power supply voltages eventually limit the excursions, and determine the maximum counting rate that can be tolerated without distortion of the output pulses. When the maximum counting rate condition is met, the preamp becomes saturated and no pulses will be output. The second limitation is that the feedback resistor $R_f$ has an intrinsic noise (Johnson noise) associated with it. The noise can be minimized by selecting a higher $R_f$ value, which is limited because

- Simple increase of $R_f \Rightarrow$ may lead to too long a time constant
- Keeping time constant by reducing $C_f \Rightarrow$ affects linearity of the preamp

The two shortcomings of the RC feedback preamp can be potentially relieved if the feedback resistor $R_f$ is eliminated. Without the feedback resistor, the pulses of charge from the detector are simply accumulated on the feedback capacitor. As shown in Fig. 6.3, the output voltage then grows in staircase pattern with each upward step corresponding to a separate pulse. Some method must be provided to reset the preamp when the staircase approaches the maximum allowable voltage. A popular way is to reset with a transistor reset circuit as in a transistor reset preamp.
B. Detector bias supply and pulse generators

Radiation detectors require the application of an external high voltage for proper operation and high voltage supplies used for this purpose are conventionally called detector bias supplies. The important characteristics of detector bias supplies are:

- The maximum voltage level and its polarity
- The maximum current available from the supply
- The degree of regulation against long-term drifts due to changes in temperature or line voltage
- The degree of filtering to eliminate ripple at power line frequency or other sources.

In the case of PMTs connected to scintillation detectors, bias supplies should cover up to 3 kV with a current of a few mA. The high voltage output must also be well regulated to prevent gain shift in the PMT. High voltage supplies for germanium semiconductor detectors may cover up to 5 kV for large size models.

An electronic pulse generator (or pulser) is always required in the initial setup and calibration of radiation spectroscopy systems. A tail pulse generator with adjustable rise and decay times is the most common and its output is fed to the test input on preamps so that the pulser output can be used for adjusting and testing shaping or timing parameters. If the output amplitude is constant with a high accuracy, the amplitude distribution measured by a pulse analysis system gives the electronic noise level present in the system. For normal pulsers, the interval between pulses is uniform and periodic. However, some tests require random generation of pulses. Randomly spaced pulses can be generated using either the noise signal from an internal component or an external trigger pulse derived from a random signal source like another radiation detector.

6.3. Pulse counting systems

A. Integral discriminator

In order to count the pulses reliably, the preamp output signal must be shaped and amplified by a shaping amplifier and then the shaped linear pulses must be converted into logic pulses. The integral discriminator is the simplest unit that does this operation and consists of a device that produces a logic output pulse only when the linear input pulse height exceeds a threshold, i.e. discriminator level. The logic output pulse is normally produced shortly after the leading edge of the linear pulse crosses the discriminator level. Integral discriminators are designed to accept shaped linear pulses in the 0 ~ 10 V range.
B. Differential discriminator (single channel analyzer)

A single channel analyzer (SCA) produces a logic output pulse only when the linear input pulse height lies between two independent discriminator levels. In some units, the lower level discriminator (LLD) and upper level discriminator (ULD) are independently adjustable while in other units, the LLD is labeled the E level and the window width or difference between levels is labeled $\Delta E$.

![Fig. 6.4. SCA principle.](image)

In normal SCAs, the time of appearance of the logic pulse is not closely coupled to the actual event timing, and therefore these logic pulses will give imprecise results when used for timing measurements. Timing SCAs are made so that the logic pulse can be much more closely correlated with the actual arrival time of the linear pulse by incorporating the time pick-off methods described later in this chapter.

C. Counter and timer

As the final step in a counting system, the logic pulses must be accumulated and their number recorded over a period of time. A counter is used for this purpose and increments one count each time a logic pulse is presented to its input.

Counters are operated in one of two modes usually: preset time or preset count. In the preset time mode, the counting period is controlled by an internal or external timer. In the preset count mode, the counter accumulates pulses until the total number of counts reaches the preset value of counts. The function of a timer is simply to start and stop the accumulation period for an electronic counter or other recording device.

6.4. Pulse height analysis systems

A. Shaping (spectroscopy) amplifier

For pulse-height or energy spectroscopy, the linear, pulse shaping amplifier performs several essential functions. Its primary role is to magnify the amplitude of the preamplifier output pulse from the mV range into the 0.1 ~ 10 V range. This makes possible accurate pulse height measurements with a peak-sensing analog-to-digital converter (ADC) or SCA. In addition, the amplifier shapes the pulses to optimize the energy resolution, and to minimize the risk of overlap between successive pulses. Most amplifiers also incorporate a baseline restorer to ensure that the baseline between pulses is held rigidly at ground potential in spite of changes in counting rate or temperature.

Frequently, the requirement to handle high counting rates is in conflict with the need for optimum energy resolution. For most radiation detectors, achieving the optimum energy resolution requires long pulse widths. On the other hand, short pulse widths are essential for high counting rates. In such cases, a compromise pulse width must be selected so that the spectroscopy system can be optimized. In this section, various techniques available for pulse shaping in the linear amplifier are described.

The linear, pulse-shaping amplifier must accept the output pulse shapes provided by the preamplifier and change them into the pulse shapes suitable for optimum energy spectroscopy.
The output for each pulse consists of a rapidly rising step, followed by a slow exponential decay. It is the amplitude of the step that represents the energy of the detected radiation.

![Preamplifier Output](image)

![Amplifier Output](image)

Fig. 6.5. Output pulse shapes from a RC feedback preamp (a) and a shaping amplifier (b).

Before amplification, the pulse-shaping amplifier must replace the long decay time of the preamp output pulse with a much shorter decay time. Otherwise, the acceptable counting rate would be seriously restricted. Fig. 6.5 demonstrates this function using the simple example of a shaping amplifier. The energy information represented by the amplitudes of the steps from the preamplifier output has been preserved, and the pulses return to baseline before the next pulse arrives. This makes it possible for a peak sensing ADC to determine the correct energy by measuring the pulse amplitude with respect to the baseline. With the shorter pulse widths at the amplifier output, much higher counting rates can be tolerated before pulse pile-up causes significant distortion in the measurement of the pulse heights above baseline.

**Delay-line pulse shaping**

Shaping amplifiers employing delay-line pulse shaping are well suited to the pulse processing requirements of scintillation detectors. The propagation delay of distributed or lumped delay lines can be combined into suitable circuits to produce an essentially rectangular output pulse from each step-function input pulse. For pulse pile-up prevention, this shaping method is close to ideal because an immediate return to baseline is obtained.

With scintillation detectors, the signal-to-noise ratio of the preamplifier and amplifier combination is seldom a limitation on the energy resolution. The energy resolution of scintillation detectors is governed by the statistics of scintillation light production and the conversion to photoelectrons at the photo cathode. However, for detectors having no internal gain, delay-line shaping is not appropriate, because the signal-to-noise ratio is inferior to that obtained with other shaping methods.

There are many circuits that can be used for delay-line shaping, and Figs. 6.6 and 6.7 are typical examples. The step pulse from the preamplifier is inverted, delayed, and added back to the original step pulse. The result is a rectangular output pulse with a width equal to the delay time of the delay line. In practice, the value of the resistor labeled $2R_D$ is made adjustable over a small portion of its nominal value to allow compensation for the exponential decay of the input pulse. With proper adjustment, the output pulse will return to baseline promptly without undershoot. The main advantage of delay line shaping is a rectangular output pulse with fast rise and fall times. In fact, the falling edge of the pulse is a delayed mirror image of the rising edge. These characteristics make delay line pulse shaping ideal for timing and pulse-shape discrimination.
applications with scintillation detectors at low or high counting rates.

By following one delay-line shaper with a second, a doubly differentiated delay-line shape is obtained, as illustrated in Fig. 6.7. The result is an output pulse with a bipolar shape. The double-delay-line (DDL) shaping is ideal for scintillation detectors with high counting rates. The baseline shift caused by high counting rates is eliminated in the bipolar shape. This benefit is gained at the expense of doubling the pulse width.

DDL shaping is often useful for simple zero-crossover timing with scintillation detectors. Double-delay-line shaping is not a good choice for detectors having a substantial preamplifier noise. Its signal-to-noise ratio is worse than single-delay-line shaping, and much worse than semi-Gaussian shaping.

CR-RC pulse shaping
The simplest concept for pulse shaping is the use of a CR circuit followed by an RC circuit. In this shaping, the preamp signal first passes through a CR shaper and then RC shaper. Fig. 6.7 shows the circuit diagrams. Although this elementary filter is rarely used, it encompasses the basic concepts essential for understanding the higher-performance, active filter techniques.

For circuit analysis, the Laplace and Fourier transformations are essential. The fundamentals of these transforms are as follows.

- **Laplace transform:** defined as 
  \[ L[f(t)] = F(s) = \int_{0}^{\infty} f(t)e^{-st} dt \]
  for a time function \( f(t) \)

  Time differentiation and integration:
  \[ L[\frac{df(t)}{dt}] = sL[f(t)] - f(0), \quad L[\int_{0}^{t} f(t')dt'] = \frac{1}{s}L[f(t)] \]

  The Laplace transforms for some functions are given at the end of the chapter.

- **Fourier transform:** 
  \[ F[f(t)] = F(\omega) = \int_{0}^{\infty} f(t)e^{-j\omega t} dt \]
  a special case of the Laplace transform!

- **Transfer function:** In general, the input-output relation of a linear circuit device is described by a differential equation

  \[ a_0V_{out}(t) + a_1 \frac{dV_{out}(t)}{dt} + \ldots + a_n \frac{d^nV_{out}(t)}{dt^n} = b_0V_{in}(t) + b_1 \frac{dV_{in}(t)}{dt} + \ldots + b_m \frac{d^mV_{in}(t)}{dt^m} \]

  \( (V_{in}(t): \text{input}, \ V_{out}(t): \text{output}) \)
Taking the Laplace transform gives
\[ V_{out}(s) = \frac{b_0 + b_1s + b_2s^2 + \cdots + b_ms^m}{a_0 + a_1s + a_2s^2 + \cdots + a_ns^n}V_{in}(s) = G(s)V_{in}(s) \quad (V_{out}(s) = L[V_{out}(t)], V_{in}(s) = L[V_{in}(t)]) \]

The function \( G(s) \) is called the **transfer function** of the device. As the Laplace transform of the delta function is “1”, the transfer function \( G(s) \) can be alternatively defined as the impulse (delta function) input response. The time domain output \( v_{out}(t) \) can be obtained through the inverse transform as
\[ v_{out}(t) = \int G(t-t')v_{in}(t')dt' \quad (G(t) = L^{-1}[G(s)]) \]

In general, the function \( G(s) \) can be converted into
\[ G(s) = \frac{(s-z_1)(s-z_2)\cdots(s-z_m)}{(s-p_1)(s-p_2)\cdots(s-p_n)} \]

The roots \( s = p_1, p_2, \ldots, p_n \) are defined as the **poles** and \( s = z_1, z_2, \ldots, z_m \) are defined as the **zeros** of \( G(s) \). For a cascade of circuit components and a system with a feedback component, the transfer functions can be obtained by
\[
\begin{align*}
G(s) &= \frac{V_{out}(s)}{V_{in}(s)} = G_1(s)G_2(s)G_3(s) \\
G(s) &= \frac{G_1(s)}{1 + G_1(s)G_2(s)}
\end{align*}
\]

![Fig. 6.8. Transfer functions for a cascade of components and a feedback case.](image)

- **Responses of the CR and RC shapers**

  1) **CR shaping**

\[
V_{in}(t) = \frac{Q(t)}{C} + V_{out}(t) \quad \Rightarrow \quad \frac{dV_{in}(t)}{dt} = \frac{i(t)}{C} + \frac{dV_{out}(t)}{dt}
\]

by \( V_{out}(t) = i(t)R \) and \( \tau = RC \), \( \tau \frac{dV_{in}(t)}{dt} = V_{out}(t) + \tau \frac{dV_{out}(t)}{dt} \)

Assuming the zero initial condition, taking Laplace transform leads to
\[
V_{out}(s) = \frac{1}{1 + \frac{s}{\tau}}V_{in}(s) = G_{CR}(s)V_{in}(s)
\]

As the signal from the preamp has a long time constant and the shaped pulse width is much smaller than the preamp time constant, we will neglect the preamp signal decay and assume the preamp signal as a step function for convenience.

For the step function input
\[
V_{in}(t) = \begin{cases} 
V_0 & (t > 0) \\
0 & (t \leq 0)
\end{cases} \quad \Rightarrow \quad V_{in}(s) = L[V_{in}(t)] = \frac{V_0}{s}
\]

the output signal becomes
\[
V_{out}(s) = \frac{\tau}{1 + \frac{s}{\tau}}V_0 \quad \Rightarrow \quad V_{out}(t) = V_0e^{-t/\tau}
\]

To figure out the noise filtering performance of the CR shaper, the transfer function in the frequency (Fourier) domain is required. This function can be obtained by replacing \( s \) with \( i\omega \) from the Laplace domain transfer function \( G_{CR}(s) \):
\[
G_{CR}(i\omega) = \frac{i\omega\tau}{1 + i\omega\tau} \quad \Rightarrow \quad |G_{CR}(i\omega)| = \frac{\omega\tau}{\sqrt{1 + \omega^2\tau^2}}
\]
2) RC shaping

\[ V_{in}(t) = i(t)R + V_{out}(t) \text{ and } i(t) = \frac{dQ(t)}{dt} = C \frac{dV_{out}(t)}{dt} \]

\[ \Rightarrow V_{in}(t) = \tau \frac{dV_{out}(t)}{dt} + V_{out}(t) \Rightarrow V_{out}(s) = \frac{1}{1 + \frac{s}{\tau}} V_{in}(s) = G_{RC}(s)V_{in}(s) \]

Output signal for the step function input:

\[ V_{out}(s) = \frac{1}{1 + \frac{s}{\tau}} V_{0} \Rightarrow V_{out}(t) = V_{0}(1 - e^{-t/\tau}) \]

Frequency domain transfer function:

\[ G_{RC}(i\omega) = \frac{1}{1 + i\omega\tau} \Rightarrow |G_{RC}(i\omega)| = \frac{1}{\sqrt{1 + \omega^2 \tau^2}} \]

![Fig. 6.9. CR, RC and CR-RC time domain responses for a step function input.](image)

![Fig. 6.10. Absolute value of the frequency domain transfer function for CR, RC and CR-RC.](image)

The time domain responses of the CR and RC shapers for a step function input are shown in Fig 6.9. By the CR filter, the decay time of the pulse is shortened. If the time constant is made sufficiently small, the output voltage is almost proportional to the time derivative of the input waveform (CR differentiator). The RC filter makes the rising edge of the pulse stretched. If the time constant is made sufficiently large, the output signal is almost proportional to the integral of the input signal (referred to as a RC integrator).

The absolute values of the frequency domain transfer functions are shown as a function of the frequency in Fig. 6.10. At the frequency \( \omega_0 = 1/\tau \), the output signal is about \( 1/\sqrt{2} \approx 0.71 \) level of the input signal for both CR and RC filters. The CR filter attenuates low frequency signals (\( \omega < \omega_0 \)) while passes the high frequency signals (high-pass filter). This improves the signal-to-noise ratio by attenuating the low frequencies, which contain a lot of noise and very little signal. The RC filter shows the opposite trend (low-pass filter). The RC filter improves the signal-to-noise ratio by attenuating high frequencies, which contain excessive noise.

To attenuate both low and high frequency noises and make the pulse shape convenient for analysis, both CR and RC filters must be employed. Based on individual transfer functions, the transfer function of the CR-RC filter is given by

\[ G_{CR-RC}(s) = \frac{1}{(1 + \frac{s}{\tau_1 s})(1 + \frac{s}{\tau_2 s})} \quad (\tau_1: \text{CR time constant, } \tau_2: \text{RC time constant}) \]

The time domain output signal for the step input is obtained as
$$V_{out}(t) = \frac{V_0 \tau_1}{\tau_1 - \tau_2} (e^{-t/\tau_1} - e^{-t/\tau_2})$$

Typically, the differentiation time constant is set equal to the integration time constant. In that case, the frequency domain transfer function become

$$G_{CR-RC}(i\omega) = \frac{i\omega \tau}{(1 + i\omega \tau)^2} \quad \Rightarrow \quad |G_{CR-RC}(i\omega)| = \frac{\omega \tau}{1 + \omega^2 \tau^2}$$

As shown in Fig. 6.10, both high and low frequency noises are efficiently filtered out by the CR-RC filter. The output response for a step input is given by

$$V_{out}(t) = \frac{V_0}{\tau} te^{-t/\tau}$$

The time constant $\tau$ is called **shaping time**. The output pulse rises slowly and reaches its maximum at $\tau$. This time interval, the time taken for the signal leading edge to rise from zero to maximum, is defined as the **peaking time**. Another conveniently used time interval is the **rise time**, which is defined as the time taken for the signal leading edge to rise from 10 to 90% of maximum.

For semiconductor detectors, the electronic noise at the preamplifier input makes a noticeable contribution to the energy resolution. This noise contribution can be minimized by choosing the appropriate amplifier shaping time constant. Fig. 6.11 shows the effect. At short shaping time constants, the series noise component of the preamplifier is dominant. This noise is typically caused by thermal noise in the channel of the field-effect transistor, which is the first amplifying stage in the preamplifier. At long shaping time constants the parallel noise component at the preamplifier input dominates. This component arises from noise sources that are effectively in parallel with the detector at the preamplifier input (e.g., detector leakage current, thermal noise in the preamplifier feedback resistor).

The total noise at any shaping time constant is the square root of the sum of the squares of the series and parallel noise contributions. Consequently, the total noise has a minimum value at the shaping time constant where the series noise is equal to the parallel noise. This time constant is called the **noise corner time constant**. The time constant for minimum noise will depend on the characteristics of the detector, the preamplifier, and the amplifier pulse shaping network.

For silicon charged-particle detectors, the minimum noise usually occurs at time constants in the range from 0.5 to 1 $\mu$s. Generally, minimum noise for semiconductor detectors is achieved at much longer time constants (in the range from 4 to 20 $\mu$s). Such long time constants impose a severe restriction on the counting rate capability. Practically, energy resolution is often compromised by selecting shorter shaping time constants in order to handle higher counting rates.

![Fig. 6.11. Optimization of the shaping time for resolution.](image)

**Pole-zero cancellation (Tail cancellation)**

Up to this point, we approximated the preamp output pulse as a step function, however, in a real preamp pulse, the falling tail is a long exponential decay instead of a simple step function. Consequently, there is a small amplitude undershoot starting at about $7\tau$. This undershoot decays...
back to baseline with the long time constant of the preamplifier. At medium to high counting rates, a substantial fraction of the amplifier output pulses will ride on the undershoot from a previous pulse. The apparent pulse amplitudes measured for these pulses will be significantly lower, which deteriorates the energy resolution.

Most shaping amplifiers incorporate a pole-zero cancellation circuit to eliminate this undershoot. The benefit of pole-zero cancellation is improved peak shapes and resolution in the energy spectrum at high counting rates. Fig. 6.12 illustrates the pole-zero cancellation circuit, and its effect. The preamplifier signal is applied to the input of the normal CR differentiator circuit in the amplifier. The output pulse from the differentiator exhibits the undesirable undershoot. To cancel the undershoot, the variable resistor \( R_{pz} \) is added in parallel with the capacitor \( C_D \), and adjusted. The result is an output pulse exhibiting a simple exponential decay to baseline with the desired differentiator time constant. This circuit is termed a **pole-zero cancellation circuit** because it uses a zero in the transfer function, expressed in Laplace transform, of the shaping circuit to cancel a pole present in the input pulse. Exact pole-zero adjustment is critical for good energy resolution. The circuit analysis can be done as follows.

By the current conservation,

\[
C \frac{d}{dt} [V_{in}(t) - V_{out}(t)] + \frac{1}{R_{pz}} [V_{in}(t) - V_{out}(t)] = \frac{1}{R} V_{out}(t) \quad (*) \quad [Cs + \frac{1}{R_{pz}} + \frac{1}{R}] V_{out}(s) = [Cs + \frac{1}{R_{pz}}] V_{in}(s)
\]

For the exponential function input \( V_{in}(s) = \frac{V_i}{s + 1/\tau_i} \), the output becomes

\[
V_{out}(s) = \frac{V_i}{s + \frac{1}{R_{pz}C}} \left( s + \frac{1}{R_{pz}C} \right) = \frac{V_i}{s + \frac{1}{\tau_i}} \left( s + \frac{1}{\tau_i} \right)
\]

The cancellation requirement leads to the condition \( R_{pz}C_D = \tau_i \).

**Semi-Gaussian pulse shaping**

If a single CR high-pass filter is followed by several stages of RC integration, the output pulse shape becomes close to Gaussian. Amplifiers shaping in this way are called **semi-Gaussian shaping amplifiers**. Its output pulse is given by

\[
V_{out}(t) \propto \frac{t^n}{\tau} e^{-t/\tau}
\]

where, \( n \) represents the number of the integrators. The peaking time in this case is equal to \( n \tau \). Hence, if the time constants are same, the peaking time of a CR-(RC)^2 circuit is twice as long as that for a simple CR-RC circuit. When the time constants are adjusted to make equal peaking times, the CR-(RC)^2 circuit gives the more symmetric shape as shown in Fig. 6.13, which makes a faster return to the baseline. The signal-to-noise ratio of the CR-(RC)^2 filter is also better.
The block diagram in Fig. 6.14 shows a more practical circuit for semi-Gaussian shaping. The integrator components, i.e. \((RC)^n\), are now replaced with active circuit components, which incorporate transistors and diodes. The function of the active filter is similar to that of the passive RC network.

**Baseline restorer**

To ensure good energy resolution and peak position stability at high counting rates, the higher-performance spectroscopy amplifiers are entirely dc-coupled (except for the CR differentiator network located close to the amplifier input). As a consequence, the dc offsets of the earliest stages of the amplifier are magnified by the amplification gain to cause a large and unstable dc offset at the amplifier output. A baseline restorer is required to remove this dc offset, and to ensure that the amplifier output pulse rides on a baseline that is tied to ground potential.

**Pile-up rejection**

When two incident particles arrive at the detector within the width of the shaping amplifier output pulse, their respective amplifier pulses pile up to form an output pulse of distorted height (Fig. 6.15). Depending on the time difference between two pulses, the pile-up pattern significantly changes as shown in Fig. 6.16. When the second pulse comes relatively late (6 \(\mu\)s and 10 \(\mu\)s shift cases) and rides on the falling tail of the first pulse, the rising edge and the height of the first pulse are not distorted, so that the first event can be processed without problem. In contrast, when the second pulse arrives relatively early (2 \(\mu\)s shift case) and rides on the rising
edge of the first pulse, the two pulses form a single pulse with a slower rising edge. In this case, both detection events are discarded. A **pile-up rejector** can be used to prevent further processing of these distorted pulses. Practically, it is implemented by adding a “fast” pulse shaping amplifier with a very short shaping time constant in parallel with the “slow” shaping amplifier (Fig. 6.17). In the fast amplifier, the signal-to-noise ratio is compromised in favor of improved pulse-pair resolving time. A fast discriminator is set above the much higher noise level at the fast amplifier output as shown in the figure (c). The falling edge of the fast discriminator output triggers an inspection interval $T_{\text{INS}}$ that covers the slow amplifier pulse width $T_W$.

If a second fast discriminator pulse from a pile-up pulse arrives during the inspection interval, an inhibit pulse is generated like (e). The inhibit pulse is used in the associated peak-sensing ADC or multichannel analyzer to prevent analysis of the piled-up event. As demonstrated in Fig. 6.18, the pile-up rejector can substantially reduce the pile-up continuum at high counting rates.

**B. Peak sensing ADC and histogramming memory**

A peak sensing analog-to-digital converter (ADC) measures the height of an analog pulse and converts that value to a digital number. The digital output is a proportional representation of the analog pulse height at the ADC input. For sequentially arriving pulses, the digital outputs from the ADC are fed to a dedicated memory, or a computer, and sorted into a histogram. This histogram represents the spectrum of input pulse heights. The dynamic range of the ADC operation is consistent with the range of the spectroscopy amplifier output, i.e. 0 ~ 10 V.

Although the peak sensing ADC is mainly used for energy spectroscopy, it can be used for time spectroscopy as well. When the output of a time-to-amplitude converter is connected to the ADC input, the histogram represents the time spectrum measured by the time-to-amplitude converter. The combination of the peak sensing ADC, the histogramming memory, and a display of the histogram forms a multichannel analyzer (MCA). If a computer is employed to display the spectrum, as usually done in these days, the combination of the ADC and the histogramming memory is called a multichannel buffer (MCB).
The operation of the Wilkinson ADC is illustrated in Figs. 6.19 and 6.20. The lower-level discriminator (LLD), is used to recognize the arrival of the amplifier output pulse. In general, the LLD is set just above the noise level to prevent the ADC from spending time analyzing noise. When the input pulse rises above the LLD, the input linear gate is open and the rundown capacitor is connected to the input. Then, the capacitor is forced to charge up so that its voltage follows that of the rising input pulse.

When the input signal has reached its maximum amplitude and begins to fall, the linear gate is closed and the capacitor is disconnected from the input. At this point, the voltage on the capacitor is equal to the maximum amplitude of the input pulse. Following detection of the input pulse peak, the rundown capacitor is disconnected from the input and connected to a constant current generator to keep a linear discharge of the capacitor voltage. At the same time, the address clock with a high frequency is connected to the address counter and the clock pulses are counted for the duration of the capacitor discharge. When the voltage on the capacitor reaches zero, the counting of the clock pulses ceases.

Since the time for linear discharge of the capacitor is proportional to the original input pulse height, the number \( N_c \) recorded in the address counter is also proportional to the pulse height. Therefore, in the Wilkinson ADC, the A-D conversion time becomes longer as the input pulse height increases. During the memory cycle, the address \( N_c \) is located in the histogramming memory, and one count is added to the contents of that location. The value \( N_c \) is usually referred to as channel number. The total number of channels is defined as conversion gain and is selectable from 256 (for low resolution applications) to 16,384 channels (for high resolution requirements).

The dead time of an MCA is composed of the ADC processing time and the memory cycle (or storage) time. For the Wilkinson ADC, the processing time is variable depending on the input pulse height as described above. The processing time is also dependent on the conversion gain and the larger conversion gains require longer processing. The processing time per channel is the...
period of the address clock. For a typical clock frequency of 100 MHz, this time is 10 ns per channel. The dead time \( \tau \) of a Wilkinson ADC is given by

\[
\tau = N_c / f_c + T_s
\]

The dead time depends on the clock frequency \( f_c \), the channel number \( N_c \) and the memory storage time \( T_s \). The storage time is 0.5 – 2 µs typically. The advantage of Wilkinson ADCs is excellent linearity (nonlinearity < 1 %). The disadvantage is the long conversion time for large A-D conversion gains.

**Successive-Approximation ADC**

The successive-approximation ADC is illustrated in Fig. 6.21. During the rise of the analog input pulse, the switch S1 is closed and the voltage on capacitor C1 tracks the rise of the input signal. When the input signal reaches maximum height, S1 is opened, leaving C1 holding the maximum voltage of the input signal. After detection of the input pulse peak, the ADC begins its measurement process.

First, the most significant bit of the digital-to-analog converter (DAC) is set. If the comparator determines that the DAC output voltage is greater than the signal amplitude \( V_s \), the most significant bit is reset. If the DAC output voltage is less than \( V_s \), the most significant bit is left in the set condition. Subsequently, the same test is made by adding the next most significant bit. This process is repeated until all bits have been tested. The bit pattern set in the register driving the DAC at the end of the test is a digital representation of the analog input pulse height. This binary number \( N_c \) is the address of the memory location to which one count is added to build the histogram representing the pulse-height spectrum. If the ADC has \( n \) bits ( \( 2^n \) channels), \( n \) test cycles are required to complete the analysis, and **this is the same for all pulse heights**.

Although successive-approximation ADCs are available with the number of bits required for high-resolution spectroscopy, their linearity is not good. This problem is overcome by adding the sliding scale linearization as shown in Fig. 6.22. For each input signal, a random analog voltage is generated and added to it before pulse height analysis. If the generated random number is \( m \), this results in the ADC reporting the analysis \( m \) channels higher than normal. By digitally subtracting the number \( m \) at the output of the ADC, the digital representation is brought back to its normal value. Due to its random nature, the added pulse averages the analysis of each input pulse height over adjacent channels (typically, 256 channels or 8-bit) in the successive approximation ADC. This improves the nonlinearity significantly ( < 1 %). The advantages of the successive-approximation ADC with sliding scale linearization are low differential nonlinearity, and a fast conversion that is independent of the pulse height.
6.5. Digital pulse processing

With current analog pulse processing systems the preamp signal from the detector is shaped, filtered and amplified by a shaping amplifier, and then digitized by a peak sensing ADC at the very end of the analog signal processing chain. In digital pulse processing (DPP) systems, the detector signal form is digitized with a sampling (or digitizing) ADC immediately after the preamplifier. The digitized signal pulse is then shaped digitally and the pulse height is extracted. The digital processor is the key element doing this operation and either a field programmable gate array (FPGA) or a digital signal processor (DSP) can be employed. After extraction of the pulse height, one count is added to the memory address corresponding to the pulse height as in analog pulse processing.

![Preamp → Sampling ADC → Digital processor](image)

Fig. 6.23. Block diagram of the digital pulse processing.

DPP allows implementation of signal filtering functions that are not possible through traditional analog signal processing. Digital filter algorithms require considerably less overall processing time, so that the resolution remains fairly constant over a large range of count rates whereas the resolution of analog systems typically degrades rather rapidly as the counting rate increases. As a result, DPP will provide a much higher throughput without significant resolution degradation. Improved system stability is another potential benefit of DPP techniques. The detector signal is digitized much earlier in the signal processing chain, which minimizes the drift and instability associated with analog signal processing.

Fig. 6.24 shows the output pulse of the preamplifier following a detection event and its digitized form. Since the signal has been digitized, it is no longer continuous. Instead it is a string of discrete values \( \{V_{in}[1], V_{in}[2], \ldots, V_{in}[i],\ldots\} \). The first step is to apply an appropriate filter as in the analog pulse processing.

![Preamp output vs. Time](image)

Fig. 6.24. Signal pulse from a preamp and its digitized form.

The simplest digital filter would be the **moving average filter**, which is defined as

\[
V_{av}[i] = \frac{1}{L} \sum_{j=0}^{L-1} V_{in}[i+j]
\]

for the \( i \)th input data point. Since this filter takes an average value for a data length \( L \), the high frequency noise component is filtered out and the output is much smoother than the original data. Fig. 6.25 shows an example of the moving average filter with \( L = 20 \).
Although the moving average filter partly attenuates the noise, its performance is not enough for pulse height analysis compared to the analog pulse shaping. The shaped pulse has a long tail and the low frequency noise components are not attenuated yet. A more popular type of the digital filter for detector pulse processing is the **trapezoidal filter**. Its principle is as follows.

For the $i^{th}$ input data point $V_{in}[i]$, 
1) Compute the average value for the next $L$ data points as shown in Fig. 6.26:

$$V_{av1}[i] = \frac{1}{L} \sum_{j=0}^{L-1} V_{in}[i + j]$$

The interval $\Delta t_L$ is the time interval corresponding to the data length $L$.

2) Make a separation gap of $G$ data points and compute another average for the data length $L$.

$$V_{av2}[i] = \frac{1}{L} \sum_{j=0}^{L-1} V_{in}[L + G + i + j]$$

The interval $\Delta t_G$ is the time interval corresponding to the data length $G$.

3) The output signal $V_{out}[i]$ corresponding to the input point $V_{in}[i]$ is computed by

$$V_{out}[i] = V_{av2}[i] - V_{av1}[i]$$

When this operation is applied to all input data points, the output pulse shape becomes trapezoidal as the name implies (Fig. 6.27). The output pulse has a peaking time equal to $\Delta t_L$, a flat top equal to $\Delta t_G$, and a symmetrical fall time equal to $\Delta t_L$. The total width of the output pulse
is $2\Delta t_L + \Delta t_G$. Hence, both $\Delta t_L$ and $\Delta t_G$ are employed as free parameters to adjust the output pulse shape as the shaping time $\tau$ is used in the semi-Gaussian analog shaping as a shaping parameter. Fig. 6.28 shows three trapezoidal filter outputs with different shaping conditions.

Fig. 6.29 compares a CR-(RC)$^2$ analog filter with $\tau = 0.5 \mu$s and an equivalent trapezoidal filter with $\Delta t_L = 1.0 \mu$s and $\Delta t_G = 0.5 \mu$s. The peaking time of the CR-(RC)$^2$ shaping is $2\tau$ and the output pulses are not symmetric, so that the total pulse width is about $6\tau$ (3 times of the peaking time). In contrast, the trapezoidal filter output pulse shows a sharp termination on completion of its total width $2\Delta t_L + \Delta t_G$. The flat top region of the trapezoidal pulse is helpful for enhancing the detector charge carrier collection when the peaking time is shorter than the collection time of a fraction of charge carriers. Moreover, the analog pulse processing chain requires another step for pulse height analysis (i.e. AD conversion) while the digitized pulse height is already available in the trapezoidal pulse. Consequently, the processing speed in DPP is faster and DPP is preferred in high counting rate measurements.

Another attractive feature of the digital pulse processing is its simplicity and flexibility for pulse timing. The arrival time or the detailed information on the rising part of a pulse is required when the pulse processing system is intended for coincidence, pulse shape analysis, particle tracking etc. In analog pulse processing, sophisticated timing modules must be set up in addition to the pulse height analysis electronics. In the digital processing, the detailed waveform of the detection signal is already digitized, so that timing operations can be implemented in the digital processor without additional units. Moreover, the algorithms for timing or pulse shape analysis can be modified easily.

![Fig. 6.28. Trapezoidal filter outputs for different shaping parameters.](image1)

![Fig. 6.29. Comparison between the CR-(RC)$^2$ and the trapezoidal filters.](image2)

There is an important fact to remember. The DPP systems are not entirely digital. An analog preamp is required to convert the detector charge to a voltage signal and some additional analog front end conditioning is required to match the preamp output signal to the input of the sampling ADC. Thus, the stability and integrity of the analog front end electronics is still important in achieving good system performance.
6.6. Pulse timing

In many applications, information on the accurate arrival time of a quantum of radiation in the detector is required. For timing information, detector pulses are usually processed quite differently than the pulse height analysis.

Time pick-off is the fundamental operation generating a logic pulse to indicate the time of occurrence of an input linear pulse. The leading edge of the logic pulse corresponds to the time of occurrence. Modular electronics doing this function are time pick-off units or triggers.

To achieve the best timing performance, an accurate time pick-off is most important. There are two cases of inaccuracy in time pick-off: **time jitter** and **amplitude walk**. Time jitter is usually induced by random fluctuations in the signal pulse size and shape. Amplitude walk is the effect induced by the variable amplitudes of input pulses. The relative importance of two categories depends on the dynamic range of the input pulse height.

**Leading edge timing**

Leading edge timing is the simplest time pick-off method and generates the output timing logic pulse when an input pulse crosses a fixed discrimination level. This method is easy to implement and is effective when the dynamic range of the input pulses is not large.

The effect of time jitter on leading edge timing is shown in Fig. 6.30. The random fluctuations can make the output logic pulse be generated at different times with respect to the centroid of the pulse. The amplitude walk in leading edge timing is illustrated in Fig. 6.31. The two pulses have identical arrival times but their output logic pulses are significantly different in timing. If this effect is too serious, the leading edge timing cannot be adopted for the applications requiring high accuracy timing.

![Fig. 6.30. Time jitter in leading edge timing.](image1)

![Fig. 6.31. Amplitude walk in leading edge timing.](image2)

Even if the input amplitude is constant, time walk can be generated when the pulse shapes of the rising part are different. This situation is usually met for germanium semiconductor detectors. In
germanium detectors, the charge carrier collection time is pretty dependent on the interaction site, which accordingly makes the pulse shape depend on the interaction position.

**Crossover timing**

If a bipolar instead of a unipolar input pulse is used for timing, the output logic pulse can be generated at the zero crossover point, which is defined as “crossover timing”. In this timing, the crossover point is independent of the pulse amplitude, so that the amplitude walk can be significantly reduced. DDL shaping is the simplest way to make a bipolar shape and therefore, is preferred in crossover timing.

**Constant fraction timing**

The principle of constant fraction timing is illustrated in Fig. 6.32. The preamp output is multiplied by the fraction $f$ that is to correspond to the intended fraction of full amplitude. The input signal is inverted and delayed for a time greater than the rise time as shown. The sum of the two wave forms makes the “Signal for timing”. The time that this pulse crosses the zero axis is $t_d + ft_r$, which is independent of the pulse amplitude and corresponds to the time at which the pulse reaches the fraction $f$ of the amplitude.

**Time spectroscopy system**

An example of a time spectroscopy system is shown in Fig. 6.33. The source emits more than two radiation quanta in cascade. The timing signal from detector 1 triggers the time-to-amplitude converter (TAC) and then the signal from the other detector defines the stop time after a proper delay. The amplitude of the TAC output is proportional to the difference between start and stop times. The pulse height is analyzed with the MCA. Thus, the accumulated spectrum represents a time spectrum. A simpler configuration can be made when a time-to-digital converter (TDC) is employed instead of the TAC.

![Fig. 6.32. Constant fraction timing principle.](image)

![Fig. 6.33. A time spectroscopy system.](image)
References

5. CAMAC ADCs, Memories, and Associated Software, ORTEC, URL: http://www.ortec-online.com/.
### Appendix 1. Laplace transforms for some functions

<table>
<thead>
<tr>
<th>Time function</th>
<th>Laplace transform</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta(t)$</td>
<td>1</td>
</tr>
<tr>
<td>Unit step $u_s(t)$</td>
<td>$\frac{1}{s}$</td>
</tr>
<tr>
<td>$t$</td>
<td>$\frac{1}{s^2}$</td>
</tr>
<tr>
<td>$\frac{t^n}{n!}$</td>
<td>$\frac{1}{s^{n+1}}$</td>
</tr>
<tr>
<td>$e^{-\alpha t}$</td>
<td>$\frac{1}{s + \alpha}$</td>
</tr>
<tr>
<td>$\sin \omega t$</td>
<td>$\frac{\omega}{s^2 + \omega^2}$</td>
</tr>
</tbody>
</table>
Problems

1. A detector preamp signal is shaped by the following circuit. For a given preamp signal form $V_{in}(t)$, Sketch the expected signal patterns at the step A and the step B. The time constants are $R_1C_1 = R_2C_2 = 2 \mu s$.

![Circuit Diagram]

2. Suppose a proportional counter is connected to the following circuit. Sketch the output signal shapes of three detection events (horizontal axis: time $t$, vertical axis: $V(t)$) and briefly explain.

3. A step function input signal (voltage $V_o$) is connected to the following circuit. Sketch the output signal shape and briefly explain the reason.
4. For a circuit given below,
(a) Write down the circuit equation between the input voltage $V_{in}(t)$ and the output $V_{out}(t)$.

(b) For the equation obtained in (a), take the Laplace transform.

(c) Using the result obtained in (b), find $V_{out}(t)$ for a step input voltage.

(d) Using the result obtained in (b), find the transfer function in the frequency domain and its magnitude. Sketch the magnitude as a function of the angular frequency.

5. (a) For a $CR-(RC)^2$ shaping amplifier, derive the 0% to 100% rise time of the output $V_{out}(t)$ in case of a step function input (amplitude $V_0$).

(b) An output pulse shape of a $CR-(RC)^2$ shaping amplifier is given. Find the shaping time constant.

6. For the following circuit, find the relation between $V_{in}(s)$ and $V_{out}(s)$.

7. The time domain output of the CR-RC shaper for a step input (amplitude $V_0$) is given by
$$V_{out}(t) = \frac{V_0 \tau_1}{\tau_1 - \tau_2} \left( e^{-t/\tau_1} - e^{-t/\tau_2} \right)$$
(a) Find the output $V_{out}(t)$ when the time constants $\tau_1$ and $\tau_2$ become identical.

(b) Find the 0% to 100% rise time for the case (a).

(c) Sketch the shape of the $V_{out}(t)$ with respect to time for a time constant of 1 $\mu$s.

8. A Wilkinson peak sensing ADC is used for a radiation spectroscopy system. The dynamic range of the ADC operation is 0 ~ 10 V.
(a) The AD conversion gain is set at 1,000 channels. For a 4 V signal, it took 2 $\mu$s for the ADC to do A-D conversion. Find the clock frequency of the ADC.

(b) The conversion gain is increased to 8,000 channels. To make this change in conversion gain, which parameter of the ADC circuit should be changed? Find the A-D conversion time of the same 4 V signal for this case.
9. Two pulses given below are fed to a successive approximation peak-sensing ADC. The dynamic range is 0-10 V.
Suppose the number of the iteration cycles is set to 3.
(a) For the signal “1”, find the ADC output for each iteration cycle.
Briefly explain the reason.
(b) For the signal “2”, find the ADC output for each iteration cycle and briefly explain the reason.

10. The signal from a preamp is digitized using a 20 MHz sampling ADC and filtered by the following algorithm:
\[ V_{out}[i] = V_{av2}[i] - V_{av1}[i], \]
\[ V_{av1}[i] = \frac{1}{L} \sum_{j=0}^{L-1} V_{in}[i+j], \]
\[ V_{av2}[i] = \frac{1}{L} \sum_{j=0}^{L-1} V_{in}[C+i+j] \]
Here, \( V_{in}[i] \) denotes \( i \)th sampled data point. An output pulse shape from this filter is given.
(a) Find the value “L”.
(b) Find the value “C”.

11. Suppose an input signal \( V_{in}(t) \) with a rise time \( t_r \) is connected to a time pickoff unit.
As shown in the figure, signal shapes (1), (2), (3) denote:
(1): \( V_{in}(t) \) is attenuated by a fraction \( f \), \( 0 < f < 1 \)
(2): \( V_{in}(t) \) is inverted and then delayed by \( t_d \).
(3): Sum of the signals “(1)” and “(2)”.
Find the time that the signal “(3)” crosses the zero axis in terms of \( f, t_r \) and \( t_d \).