NEUTRON INTERACTIONS

1. Introduction

The neutron is indirectly ionizing like the photon. It is unstable when free and beta decays to the proton with an 11.7 m half-life. Unlike the photon, interactions with the atomic electrons are not significant so that only neutron-nucleus reactions need be considered. These may be classified as elastic and non-elastic interactions. The latter are further subdivided into inelastic scattering, capture reactions in which the neutron combines with the target which then decays by the emission of a γ-ray or charged particle and (n,2n) reactions. The type and nature of the reaction varies with neutron energy. Elastic scattering and radiative capture reactions have no threshold for any target nucleus and are therefore dominant in the low energy region.

In describing nuclear reactions in general it is common to use a partial wave expansion based upon the following simplified considerations. If the incident particle has momentum \( p \) and approaches the target with impact parameter \( b \) then the relative angular momentum is \( bp = \hbar k \). Since the nuclear force is short range there is an impact parameter such that the neutron will never interact with the target. This will roughly occur for \( b > R \) where \( R \) is the nuclear radius. The maximum angular momentum involved in the interaction will then be \( \ell_{\text{max}} = kR \) in units of \( \hbar \). For low energies such that \( kR < 1 \) only \( \ell = 0 \), or s-wave interactions, occur with significant probability. As the energy increases waves with successively higher angular momentum contribute to the cross section.

As the energy increases from zero the total cross section for most targets exhibits dramatic fluctuations referred to as resonances. At the resonance energy the cross section is maximum and has a sharply peaked structure. The width of the peak at half-maximum is the resonance width and designated \( \Gamma \). Further increase in energy above the resonant region, which typically lies below a few hundred kilovolts, leads to a relatively smooth cross section. Averaging the cross section over energy bins large enough to contain several resonances reveals the gross energy behaviour.

2. Elastic scattering

The differential cross section for elastic scattering is conveniently given as the square of the scattering amplitude

\[
f(\theta) = \frac{1}{2ik} \sum_{\ell=0}^{\ell_{\text{max}}} (2\ell + 1) \cdot (e^{2i\delta_\ell} - 1) \cdot P_\ell(\cos \theta) \tag{1}
\]

where \( \delta_\ell \) is the phase shift for the partial wave of angular momentum \( \ell \). The set of phase shifts determines the specific cross section. At sufficiently low energies so that only s-wave interactions are significant the scattering amplitude is determined solely by the \( \ell = 0 \) term and the scattering is isotropic in the center of mass. In this same region the average elastic scattering cross section is energy independent. For hydrogen the cross section for this process is 20 b in the low energy region dropping to 2 b at 4 MeV.
As the energy is increased more partial waves corresponding to higher relative angular momenta contribute significantly to the cross section. The process is no longer isotropic and eventually becomes sharply forward directed.

The differential cross section exhibits a diffraction pattern with small sub-maxima at angles of constructive interference. This is indicated in Fig.1. As a rough measure the scattering is significant for angles less than \( \lambda^{-1}/R \) where \( \lambda \) is the reduced wave length and \( R \) is the nuclear radius. For 10 MeV \( \lambda^{-1}=1.4 \) fm while for Fe the radius is approximately 4 fm giving an angle of about 18°.

In the low energy region from thermal to a few hundred keV for light nuclei or a few keV for medium to heavy nuclei the cross section exhibits resonances. The resonance width is the sum of partial widths designated \( \Gamma_n \) for neutron decay and \( \Gamma_\gamma \) for radiative decay. In rare cases where charged particle decay is allowed such as the \(^{14}\text{N}(\text{n},p)^{14}\text{C}\) and \(^{10}\text{B}(\text{n},\alpha)^7\text{Li}\) reactions then corresponding charged particle widths are also included.

Resonance elastic scattering corresponds to neutron decay of the compound state formed when the target nucleus captures a neutron. The capture and decay are in fact reverse processes. Just as the incoming neutron may in general carry orbital angular momentum, so may the outgoing neutron. In the resonance energy region \( s \)-wave interactions are dominant, although much weaker \( p \)-wave resonances have been observed. Discussion here will be restricted to \( s \)-wave resonances. The neutron width for \( s \)-wave resonances is energy dependent and may be written \( \Gamma_n = \Gamma_n^0 \sqrt{E/E_0} \) where \( E_0=1 \) eV and \( \Gamma_n^0 \) is the reduced width. The resonance corresponds to a multiparticle excitation with energy greater than the neutron binding energy and is characterized by a specific total angular momentum and parity \( J_\tau^\pi \).

Because of the neutron spin, the resonance angular momentum is one of \( J_\tau \pm \frac{1}{2} \) where \( J_\tau \) is the ground state angular momentum of the target nucleus. The parity is the same as that of the target ground state.

Very near the resonance energy \( E_r \) the scattering cross section \( \sigma_{nn}(E) \) may be approximated by

\[
\sigma_{nn}(E) = \frac{\pi}{k^2} \frac{g_r \Gamma_n^2}{(E_r-E)^2 + \Gamma_n^2/4} \frac{\Gamma_n}{\Gamma} \sigma_n(E)
\]

where the last expression indicates that it is the product of the total neutron cross section with the probability for neutron decay. The statistical factor \( g_r \) is the ratio of the number of final states \( 2J_\tau + 1 \) to the number of initial states \( 2(2J_\tau + 1) \). The integral over energy, important in evaluating the average cross section is then

\[
\int_0^\infty \sigma_{nn}(E)dE = \frac{2\pi^2}{k^2} g_r \Gamma_n \frac{\Gamma_n}{\Gamma}
\]
The general situation is more complicated however. At any energy the cross section results from contributions of all the resonances together with direct potential scattering, all of which appear as amplitudes. This may be written as

\[ \sigma_{mn}^\pm (E) = \frac{\pi}{k^2} g_n |a^\pm| \sum_{j=1}^{\infty} \frac{\Gamma_j}{(E_j - E) + i\Gamma_j/2} \]  

for each spin state, designated + for the higher and - for the lower possibility and it is easy to show that \( g_n + g_r = 1 \). The quantity \( a^\pm \) is the amplitude for potential scattering. One consequence of this is the appearance of potential-resonance interference effects leading to asymmetry in the resonance shape. Because of the increase in neutron width with energy, generally the neutron width dominates and to a good approximation the total width and neutron width become equal. The average scattering cross section may be written

\[ \overline{\sigma}_{nn}(E) = \frac{1}{\Delta E} \int_{E}^{E+\Delta E} \left( \sigma_{nn}^+(E') + \sigma_{nn}^-(E') \right) dE' \]  

The range of integration, \( \Delta E \) is taken to be large compared to the width and spacing of the resonances, but small compared to \( E \). Under these conditions the contributions from all cross terms to the integrals vanish. The potential term \( \pi a^2/k^2 \) may be written \( \pi R^2 \), where \( R \) acts as an effective interaction radius and is referred to as the potential scattering radius.

**Figure 2** Variation of potential scattering radius with mass number. The curve is \( 1.35 A^{1/3} \). 

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The variation of the $R'$ with mass number is shown in Fig. 2. The potential scattering length exhibits strong oscillations about the curve $1.35A^{1/3}$, roughly the geometrical value. The value is greater than the geometrical value by a factor of two at $A=10$, and by about 1.3 for $A$ between 50 and 100. It dips to about 0.7 and 0.6 of the geometrical value at $A=40$ and 140.

Each resonance term in the region between $E$ and $E+\Delta E$ contributes an area term as given in Eqn(3) with $\Gamma_n \approx \Gamma$ making the last term unity. Then the average cross section can be written

$$\bar{\sigma}_{nn}(E) = \pi R'^2 + \frac{2\pi^2}{k^2} \frac{1}{\Delta E} \sum_{j=1}^{N} g_j \Gamma_{n_j}$$  \hspace{1cm} (6)

The last term in the above equation can be rewritten in terms of reduced widths all calculated at $E$ to give the strength function

$$S^0 = \frac{1}{\Delta E} \sum_{j=1}^{N} g_j \Gamma_{n_j} \Gamma_0$$  \hspace{1cm} (7)

Since $\Delta E/N$ is the average spacing between resonances, equivalent to the inverse of the level density, then essentially the strength function is the ratio of the average reduced neutron width to the spacing. The strength function, or more properly the s-wave strength function, like $R'$, exhibits a systematic dependence on mass number. The strength function exhibits broad maxima at approximately $A=50$, 145 and 180 with peak values of roughly $5 \times 10^{-4}$, widths of some 10 mass units and valleys of roughly an order of magnitude lower.

\textbf{Figure 3} Variation of strength function with $A$

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The average cross section becomes

$$\bar{\sigma}_{nn}(E) = \pi R^2 + \frac{2\pi^2}{k^2} S_0 \sqrt{E}$$  \hspace{1cm} (8)$$

The neutron strength function is analogous to the oscillator strength function used to describe photon and charged-particle interactions. Because in this case the strong interaction is involved it is not possible to use perturbation theory. Reasonable success has been had in reproducing the observed strength function by considering the case of the neutron interacting with a square well.

3. Radiative capture

In the radiative capture or (n,γ) reaction a neutron combines with a target of mass number A to produce a compound state in the product nucleus of mass number A+1. The excitation energy of this state is equal to the sum of the neutron separation energy and the kinetic energy of the neutron. The radiative capture cross section varies inversely with velocity in the s-wave region and consequently is most important at low energies. In this region the neutron separation energy is much greater than the kinetic energy.

In the most important case the neutron energy is .025 eV on average corresponding to the thermal region. Typically the neutron separation energy is about 8 MeV and in radiative capture this energy is released in the form of one or more gamma-rays. The compound state may decay to the product ground state in which case the entire energy is given to a single gamma-ray. For the H(n,γ)D reaction this is the only possible decay mode since the deuteron has no excited states, and the reaction results in the emission of 2.22 MeV photons. For the 14N(n,γ)15N reaction the separation energy is 10.8 MeV. The 15N nucleus has several excited states in the region below the separation energy to which the compound state may decay. The result is a spectrum of γ-rays, as shown in Fig. 2, ranging up to 10.8 MeV consisting of both the primary transitions from the compound state and secondary transitions corresponding to de-excitation of levels populated in the primary decay. In heavy nuclei where the average level spacing may be only of the order of 0.1 MeV some 80 levels occur in the range up to 8 MeV so that the capture γ-ray spectrum is complicated. The average number of γ-rays released in neutron capture is approximately 3 for typical situations.

The radiative cross section and the corresponding radiative width Γγ are both the sum of a large number of partial contributions from the many primary decays. As a result while interference effects to occur for the

Figure 4. Thermal neutron radiative capture γ-ray spectrum for a mixed N-Be target.
partial cross section for a single primary transition this effect is averaged out for the total radiative cross section. Consequently the cross section becomes the incoherent sum

$$\sigma_{n\gamma}(E) = \pi \sum_{j=1}^{\infty} \frac{g_j \Gamma_{n_j} \Gamma_{\gamma_j}}{(E-E_j)^2 + \Gamma_j^2/4}$$

(9)

The average value is of less interest since the radiative capture process is generally negligible with respect to elastic scattering at higher energies where the resonances begin to overlap. The low energy region, where $E \ll E_j$ and $E_j \gg \Gamma_j$, is of interest. Here Eqn(9) under the preceding conditions becomes

$$\sigma_{n\gamma}(E) = \frac{\pi \sqrt{E}}{k^2} \sum_{j=1}^{\infty} \Gamma_{n_j} \Gamma_{\gamma_j}/E_j^2$$

(10)

Since $E = \hbar^2 k^2/2\mu$, where $\mu$ is the reduced mass, approximately $A m/(A+1)$ for target mass number $A$, the cross section is seen to vary inversely with the square root of the energy or as $1/v$, where $v$ is the relative velocity. While the “$1/v$” cross section is commonly observed departures occur when the conditions stated above are not met. This most commonly occurs when a resonance is located at very low energies near the thermal energy, so there is no region satisfying $E \ll E_j$. An interesting case is that of a negative resonance corresponding to a state located just below the neutron separation energy, but with a width broad enough to influence the continuum behaviour.

4. Inelastic Scattering

In the inelastic or $(n,n')$ reaction the nucleus is left in an excited state after the scattering and the scattered neutron energy in the center-of-mass system is less than the incident energy by the excitation energy. The excited state decays by $\gamma$-emission leading to an inelastic scattering $\gamma$-ray spectrum. The threshold for this reaction is the energy of the first excited state. This may be quite high in light nuclei, being 4.5 MeV and 6.1 MeV in $^{12}$C and $^{16}$O respectively. While the cross sections for thermal radiative capture are often in the region of 10 to 1000 b the values for $(n,n')$ reactions are of the order of 0.1 b. The cross section rises abruptly with energy above the threshold approximately varying as $(E-E_\chi)^{1/2}$ where $E_\chi$ is the excitation energy.

5. Charged particle production

Neutron reactions with charged particle exit channels are normally characterized as threshold reactions. There are however four important exceptions. The first of these is the $^{14}$N$(n,p)^{14}$C reaction which has a Q-value of -0.62 MeV and a thermal cross section of 1.5 b. This reaction is
responsible for a significant contribution to the dose resulting from thermal neutron exposure. It is also the mechanism for the production of cosmogenic $^{14}\text{C}$ in the upper atmosphere. The second and third reactions are $(n,\alpha)$ reactions on $^6\text{Li}$ and $^{10}\text{B}$ which have Q-values of -4.5 MeV and -2.7 MeV respectively. Finally the $(n,p)$ reaction on $^3\text{He}$ with a Q-value of -7.7 MeV is also notable. The last three reactions are used in the detection of thermal and low energy neutrons. This is accomplished by doping scintillator material with $^6\text{Li}$ or $^{10}\text{B}$ or else by using $^3\text{He}$ or $^{10}\text{BF}_3$ as a filling gas in proportional counters.

A summary of threshold values for the three major complex nuclei in tissue is given in table 1. In contrast to $(n,n')$ and $(n,2n)$ reactions, the cross section for charged particle exit channels rises gradually from threshold since the particle must tunnel through the Coulomb barrier.

Table 1. Thresholds for neutron-induced reactions (MeV)

<table>
<thead>
<tr>
<th>Target</th>
<th>Exit Channel</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>n'</td>
</tr>
<tr>
<td>$^{12}\text{C}$</td>
<td>4.5</td>
</tr>
<tr>
<td>$^{14}\text{N}$</td>
<td>2.3</td>
</tr>
<tr>
<td>$^{16}\text{O}$</td>
<td>6.1</td>
</tr>
</tbody>
</table>

6. Neutron moderation

When neutrons undergo elastic collisions energy is lost to the target and the neutron slows down. The energy losses are a relatively large fraction of the initial energy unlike heavy charged particles however so that a C.S.D.A. approach is not suitable. Instead the quantity lethargy corresponding to $\ln(E_0/E)$ is introduced. As discussed in section 2 the probability density for the scattered energy in the isotropic scattering regime is

$$p(E) = \frac{1}{2\alpha E_0} (1-2\alpha) E_0 \leq E \leq E_0$$ \hspace{1cm} (11)$$

where $\alpha = 2mM/(M+m)^2$ as before. Defining $x = E/E_0$ the average lethargy after a collision is

$$\zeta = <\ln(E_0/E)> = \int_1^{1-2\alpha} \ln x dx/2\alpha$$

$$= 1 + (1-2\alpha) \ln(1-2\alpha)/2\alpha$$ \hspace{1cm} (12)$$

Note that the lethargy increase is independent of energy. After $n$ collisions the increase is $n\zeta = \ln(E_0/E_f)$ where $E_f$ represents the final energy reached. The number of collisions required to reach a final energy on average is then given by

$$n = \frac{1}{\zeta} \ln(E_0/E_f)$$ \hspace{1cm} (13)$$

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In neutron physics it is customary to refer to the interaction probability per unit path length as the macroscopic cross section, designated $\Sigma$. The increase in lethargy with path length is

$$\frac{du}{dx} = \xi \Sigma_s$$  \hspace{1cm} (14)

where $\Sigma_s$ is the macroscopic cross section for scattering. Then the equivalent of stopping power is obtained from the fact that $du/dx$ is $-E^{-1}dE/dx$ so that

$$S_n = \xi \Sigma_s E$$  \hspace{1cm} (15)

The quantity equivalent to range is the path length required for a neutron of initial energy $E_0$ to slow down to thermal energy $E_{th} = 0.025$ eV. From (5)

$$R_n = \frac{\ln(E_0/E_{th})}{\xi \Sigma_s}$$  \hspace{1cm} (16)

It is worth pointing out that the conventional stopping power for this process is $S = \alpha \Sigma_s E_0$. For $M \gg m$ so that $\alpha \ll 1$ then $\xi = \alpha$ and the two descriptions become identical when the average fractional energy loss is small. Because of the large scattering angles involved the range given in Equ.(7) is not a useful quantity.
PROBLEMS FOR DISCUSSION
(NEUTRON INTERACTIONS)

1. (a) Find $\ell_{\text{max}}$ for an 11 MeV neutron incident upon a nucleus of radius 4 fm. (Use $\hbar c = 197$ MeV\(\cdot\)fm)
(b) What would be the maximum power of $\cos \theta$ appearing in the differential cross-section for elastic scattering in this case?

2. What are the advantages and disadvantages of using the neutron capture reaction to produce radioisotopes? The reasons are beyond the scope of the notes, but are worth discussing.

3. (a) Find an approximate expression for the average lethargy per collision, $\xi$ in terms of the neutron mass $m$ and the target mass $M$ for the case $M \gg m$.
(b) For the same approximation, find the relation between $\xi$ and the average energy loss per collision.